

# Introductory Biomechanics

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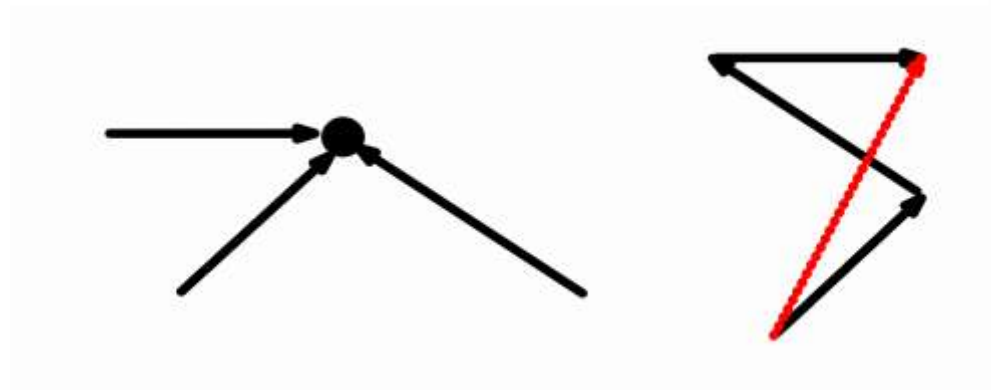
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# The Basics

- Overall objective is an overview of the underlying principles of biomechanics with relevant examples
- We will discuss:
  - Equilibrium of forces and moments
  - Kinematics of joints
  - Degrees of Freedom
  - Determinacy
  - A clinically relevant application of biomechanical modelling

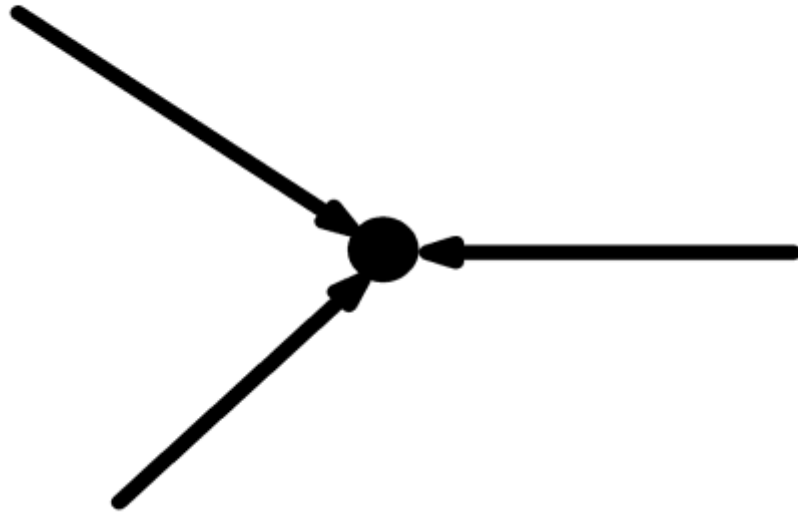
# Vectors and Resultants

**Forces are vectors – they have direction and magnitude**  
**They can be added head to tail – resultant from first tail to last head**



If a set of forces act together at a point, this is equivalent – in both magnitude and direction to the vector sum of the applied forces

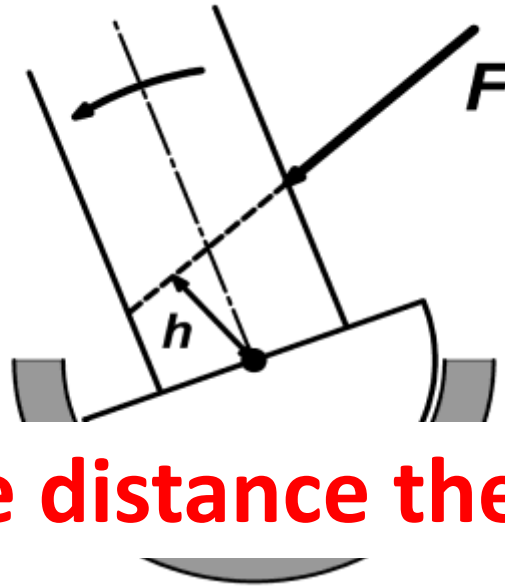
# Equilibrium



If the vector diagram produces a closed figure then the resultant is zero. This is

**EQUILIBRIUM**

# Moments

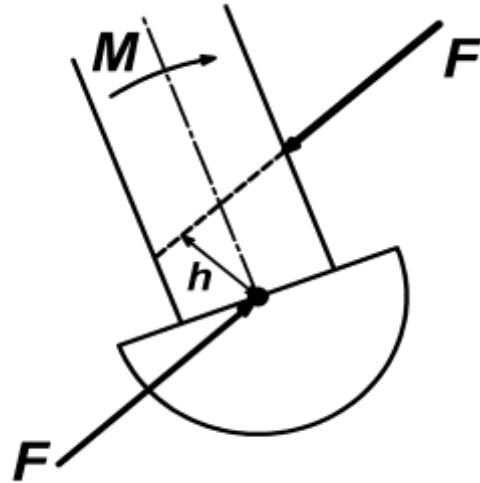


**N.B The smaller the distance the bigger the force!**

A **MOMENT** about a point is defined as the product of the force \* the perpendicular distance

$$M = F * h$$

# Equilibrium of a Joint – forces and moments



Force equilibrium achieved by a pair of equal and opposite forces  $F$

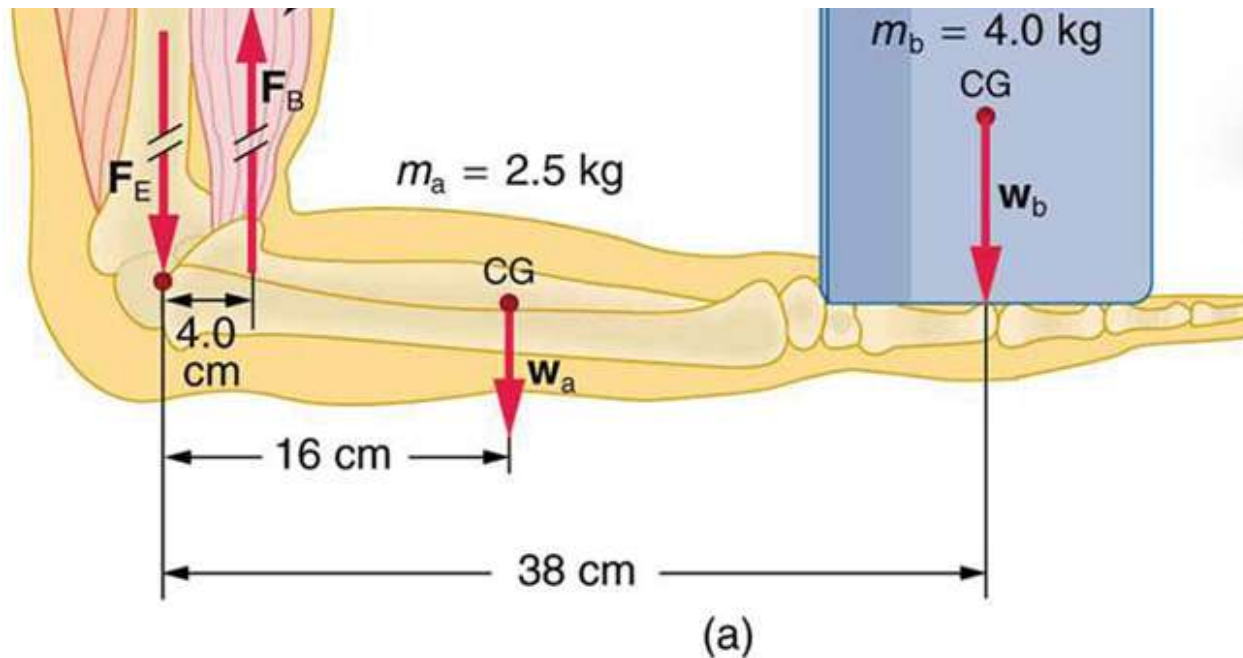
Rotational equilibrium achieved by a applied moment  $M$  opposing **COUPLE** produced by pair of forces separated by distance  $h$

# Mass, Weight and Force

- According to Newton's Laws (see later), the acceleration of a body is equal to its mass \* the acceleration.
- The units of force are **Newtons**
- If a body falls under gravity it will fall at the acceleration due to gravity ( $g = 9.81 \text{ m/s}^2$ ) and is subjected to a force = mass\*9.81.
- This force is defined as the weight =  $W = mg$   
Newtons

# Equilibrium at the Elbow

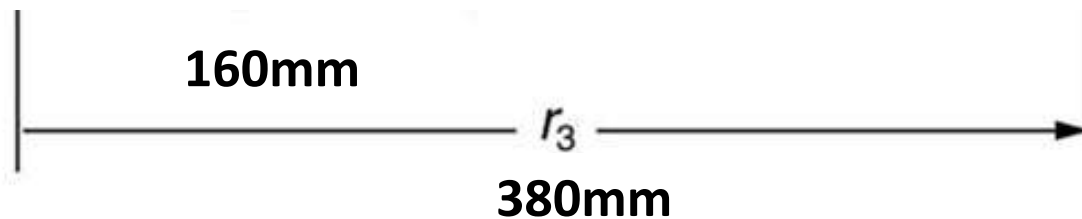
For equilibrium, the elbow extension moment resulting from the forearm weight and the weight of the book must be counterbalanced by the biceps tension



# Free Body Diagram



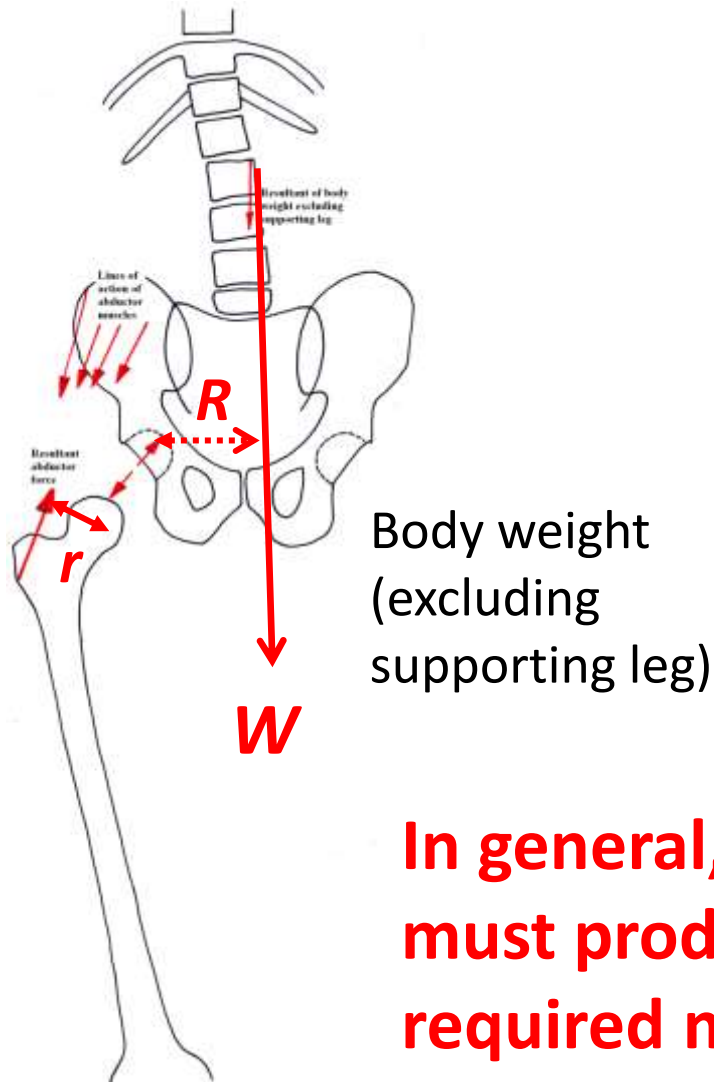
**N.B. Joint and Muscle Forces much greater than supported load!**



To find  $F_b$ : Take moments about O:  $F_b \cdot 40 = (2.5 \cdot 9.81 \cdot 160) + (4 \cdot 9.81 \cdot 380)$   
 $F_b = (3924 + 14911) / 40 = 471\text{N}$

Joint Force  $F_e = F_b - w_a - w_b = 471 - 25 - 40 = 406\text{N}$

# Standing on one Leg – why is joint force greater than body weight?



Distance of weight vector from joint centre  $R$  much greater than distance of muscle vector

$$\text{Muscle force} * r = W * R$$

$$\text{Muscle force } FM \gg W$$

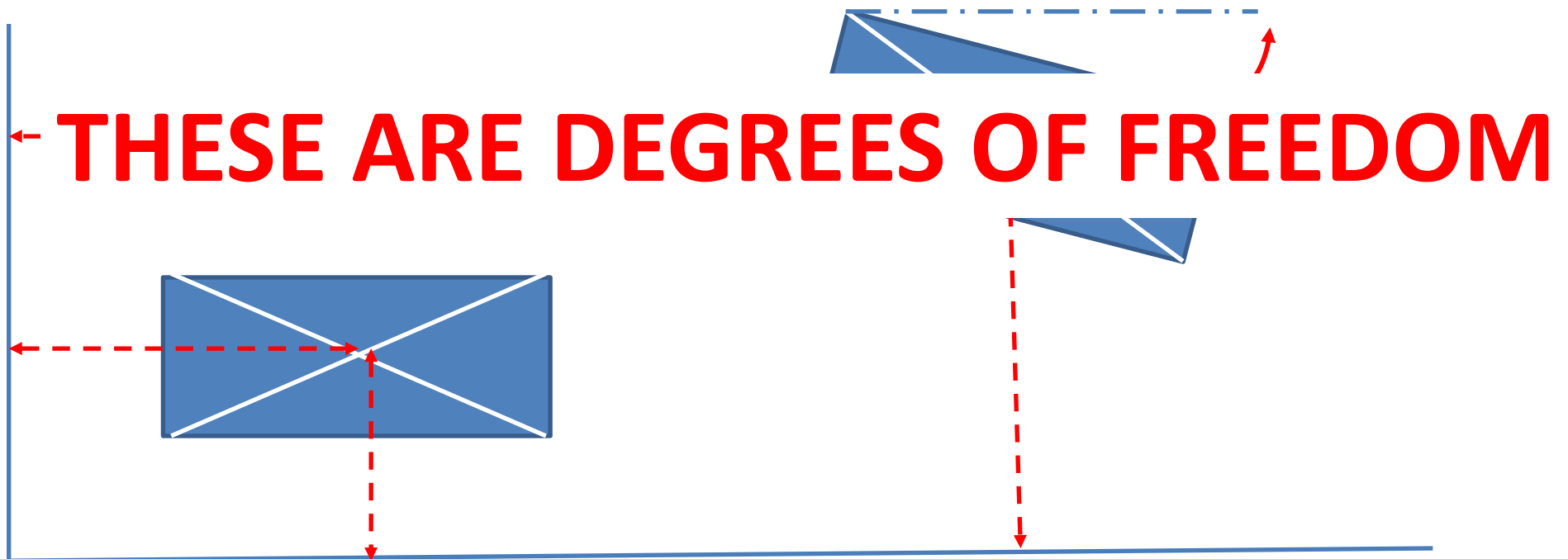
$$\text{Joint force} = \text{vector sum of } FM + W$$

Body weight  
(excluding  
supporting leg)

**In general, muscles lie close to joints and so must produce large forces to achieve required moments**

# Degrees of Freedom and Constraints

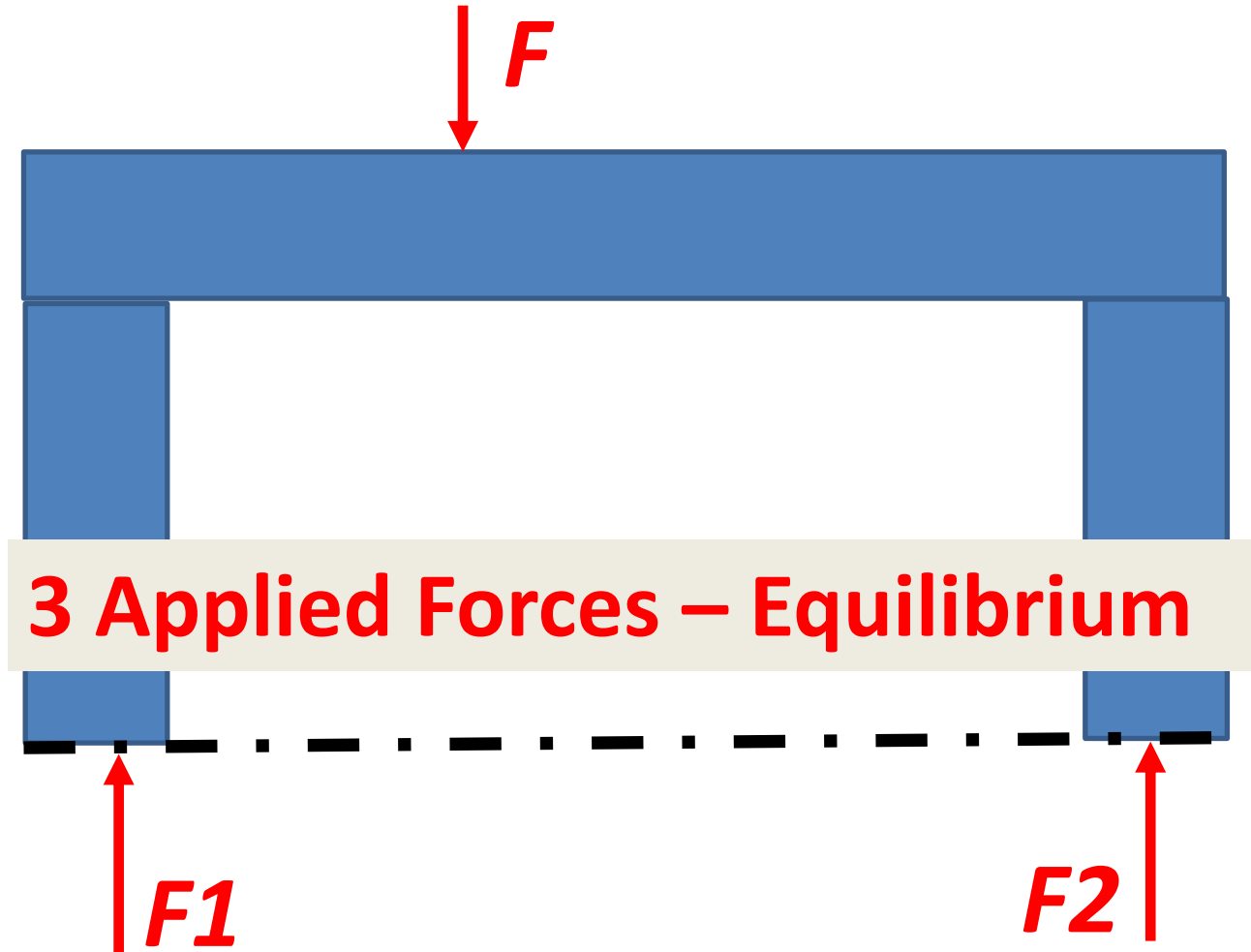
In two dimensions, the position of a rigid body requires three pieces of information - coordinates  
Two distances and one angle



# Degrees of Freedom and Constraints

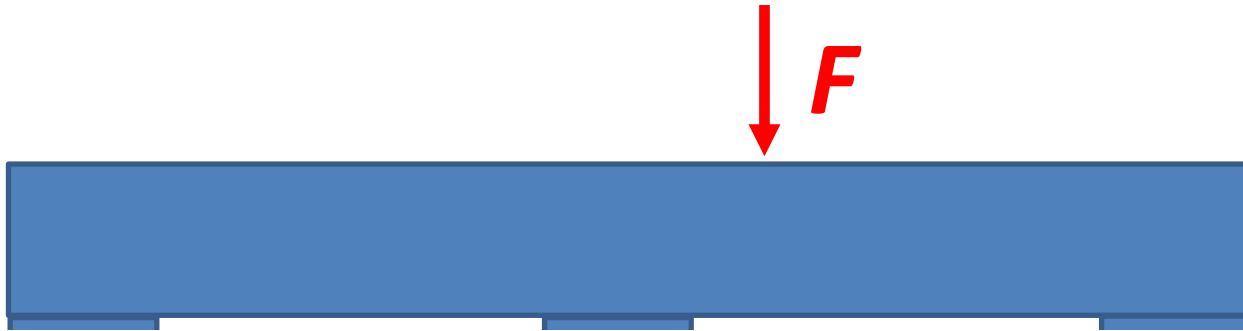
- In 2D space an object has 3 degrees of freedom – require 3 coordinates to define its position
- In 3D space, we require 6 coordinates – 3 distances + 3 angles
- Any connection of a body imposes a CONSTRAINT and reduces the number of degrees of freedom

# Degrees of Freedom and Constraints

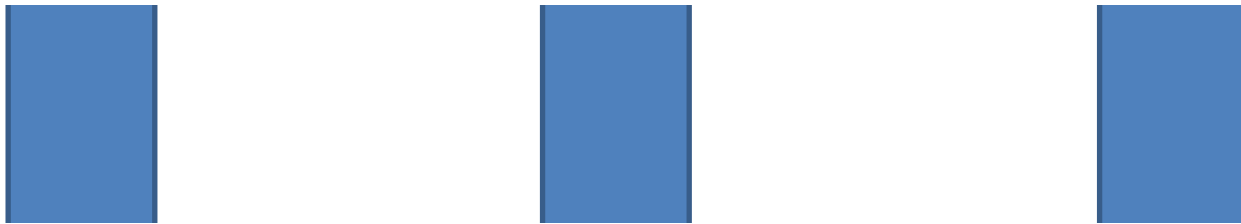


**No of applied forces= number of degrees of freedom**

# Three legs - overconstrained

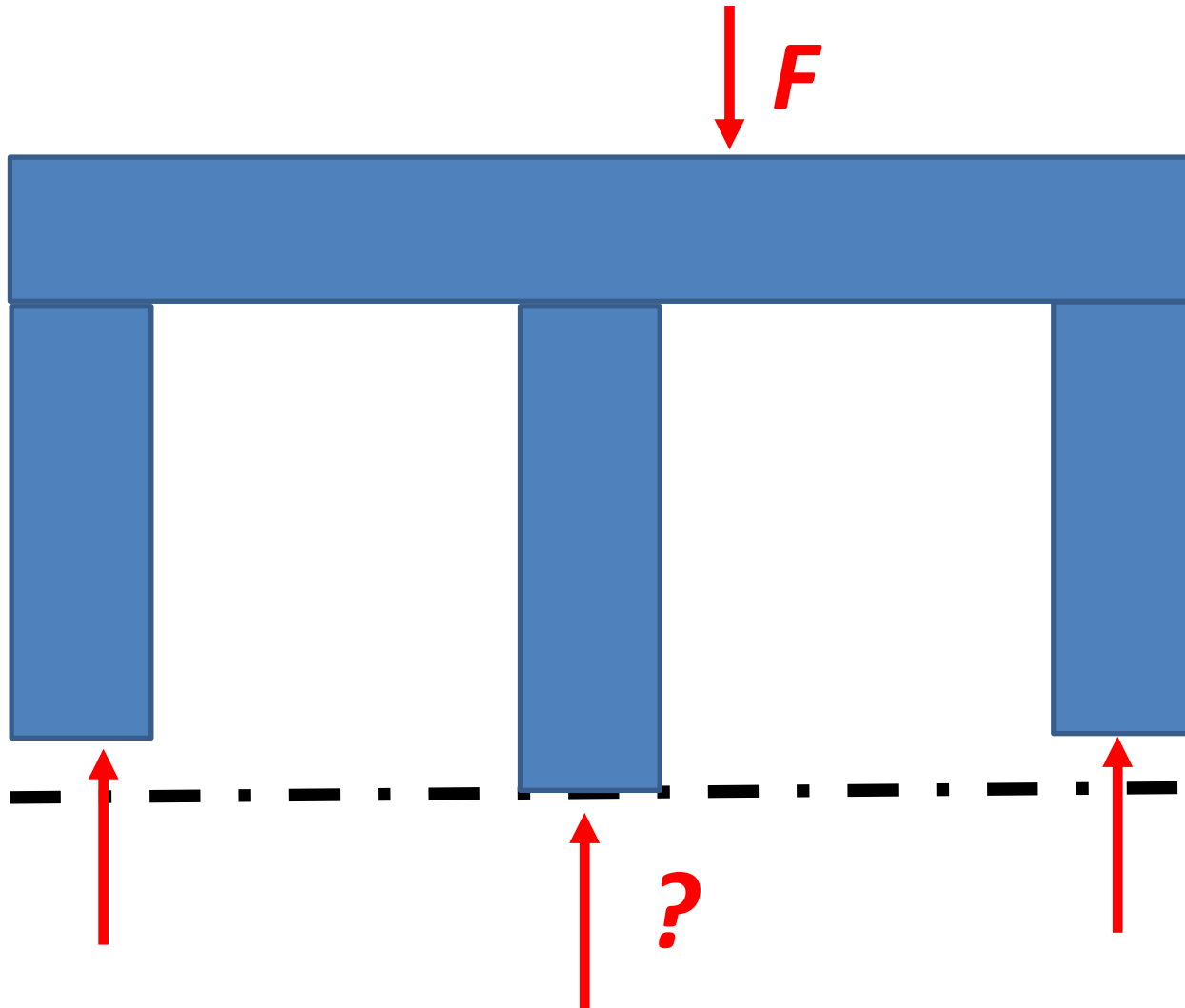


**How can we calculate the loads?**

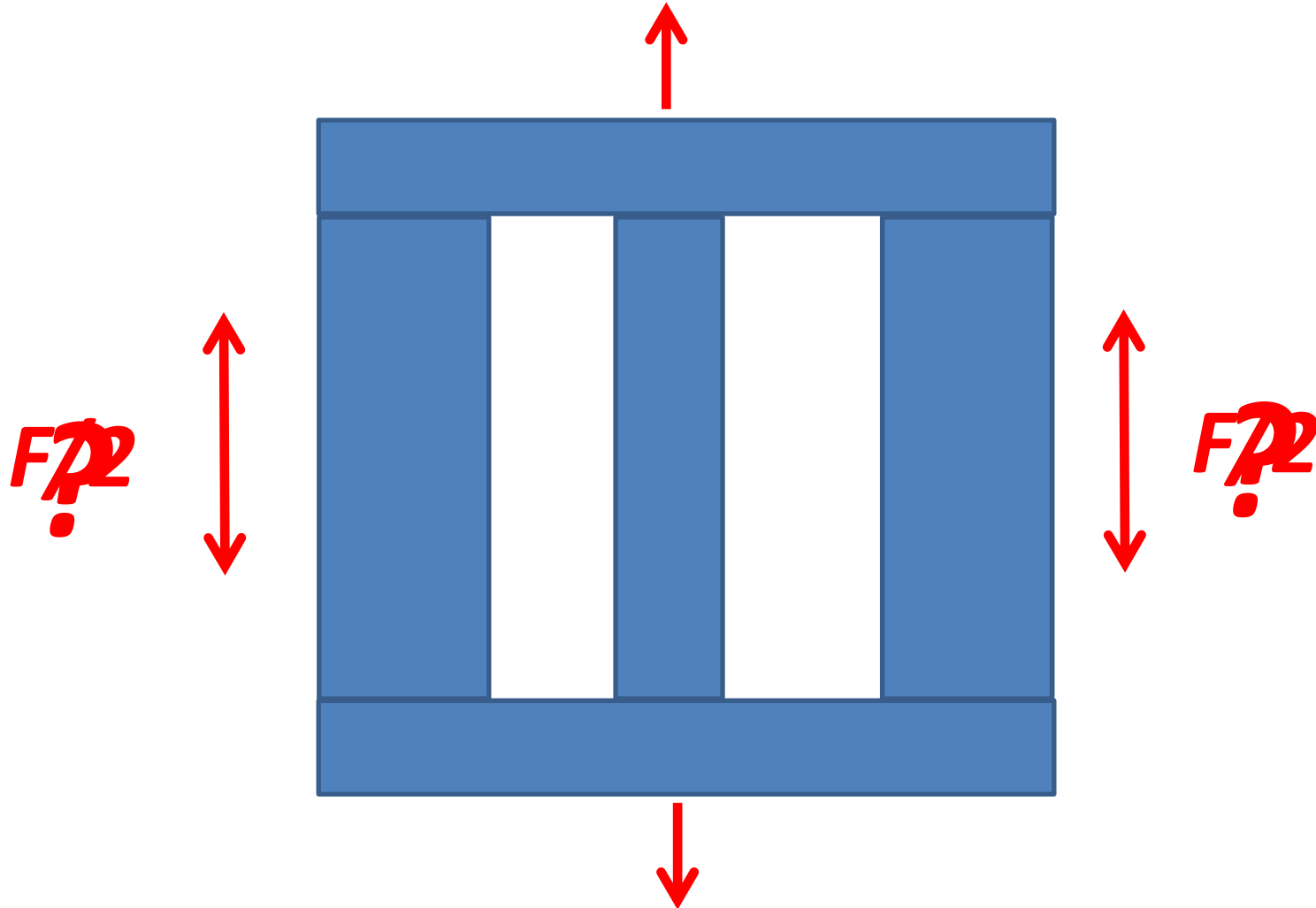


**This is a *stress analysis* problem where the load transmission is influenced by material properties and geometry**

Wobbly table – what are the support forces?

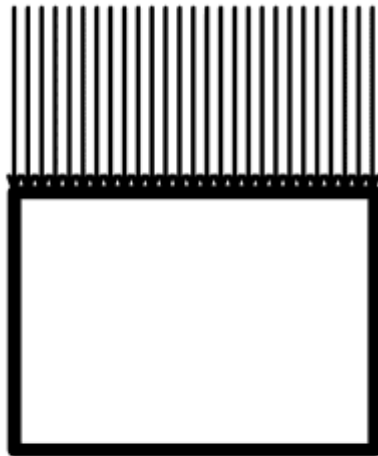


Largest force passes through stiffest element

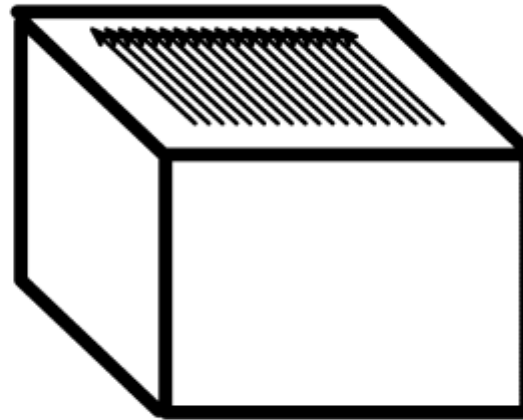


# Forces and Stresses

**Stress may be defined as the intensity of force = force per unit area: Units of stress =  $\text{N/m}^2$  = Pascals**

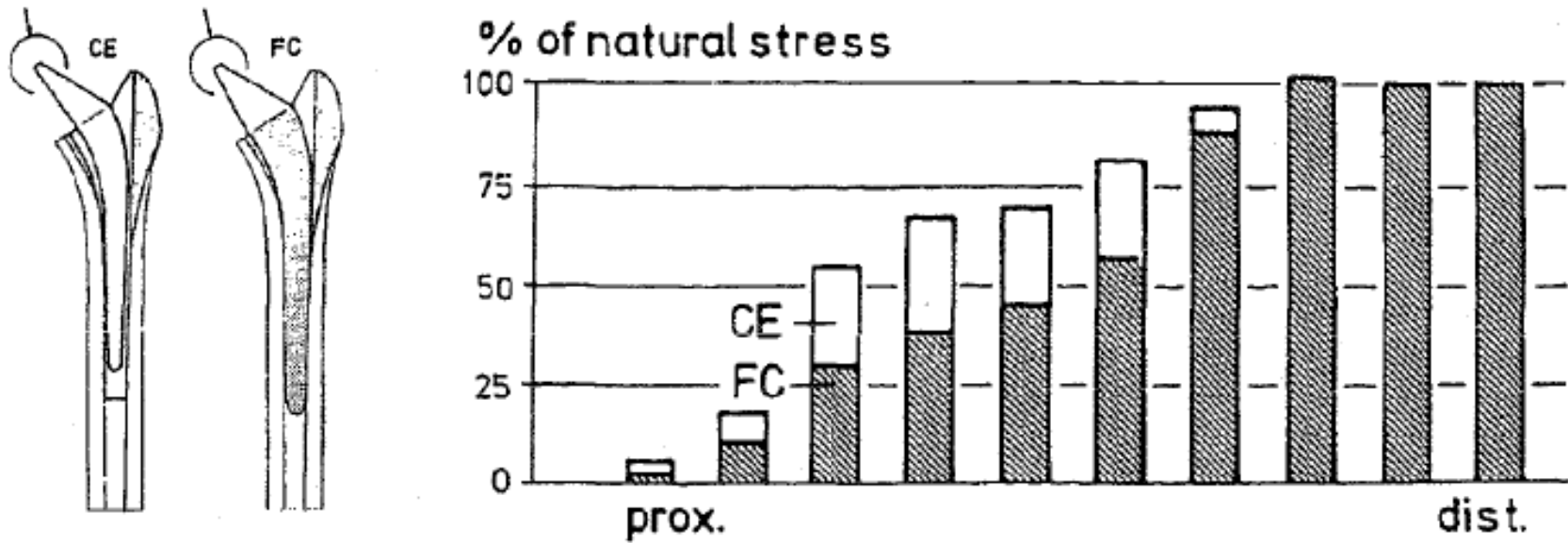


**Normal stress**



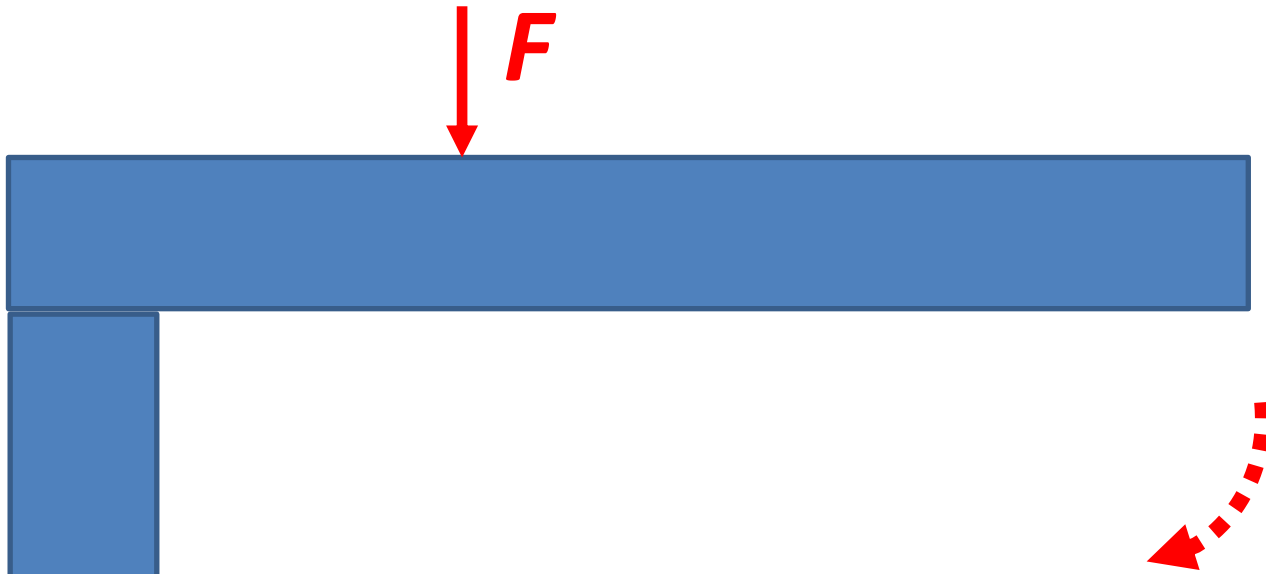
**Shear stress**

# Stress Shielding – Huiskes (1992)



*Fig. 1.* — An illustration of 'stress shielding' around a cemented (CE) and a noncemented (FC) femoral stem. The cortical bone stresses are shown, in each case, as a percentage of the 'natural' case for the same hip-joint load if the stem were not present. The difference between natural and actual is the extent of stress shielding (13).

# Underconstrained!



**Dynamics – motion results from applied loads**

| ' .

# Summary of Degrees of Freedom and Constraints

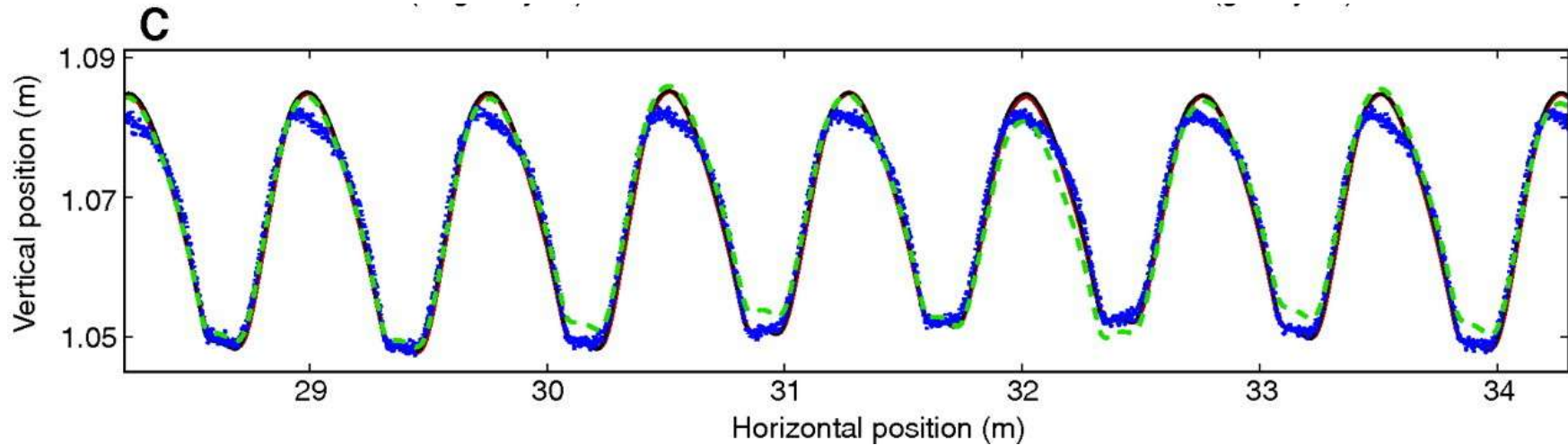
- No of DoF  $>$  No of Constraints = Mechanism (Dynamics)
- No of DoF = No of Constraints = Determinate structure (Statics)
- No of DoF  $<$  No of constraints = Indeterminate structure (stress analysis)

# Dynamics – Newton's Laws

## Relationships between force and acceleration

- I: If the vector sum of all the forces is zero, then there can be no resulting acceleration
- II: The acceleration of a body is proportional to the applied force and in the same direction
- **Force = mass \* acceleration**

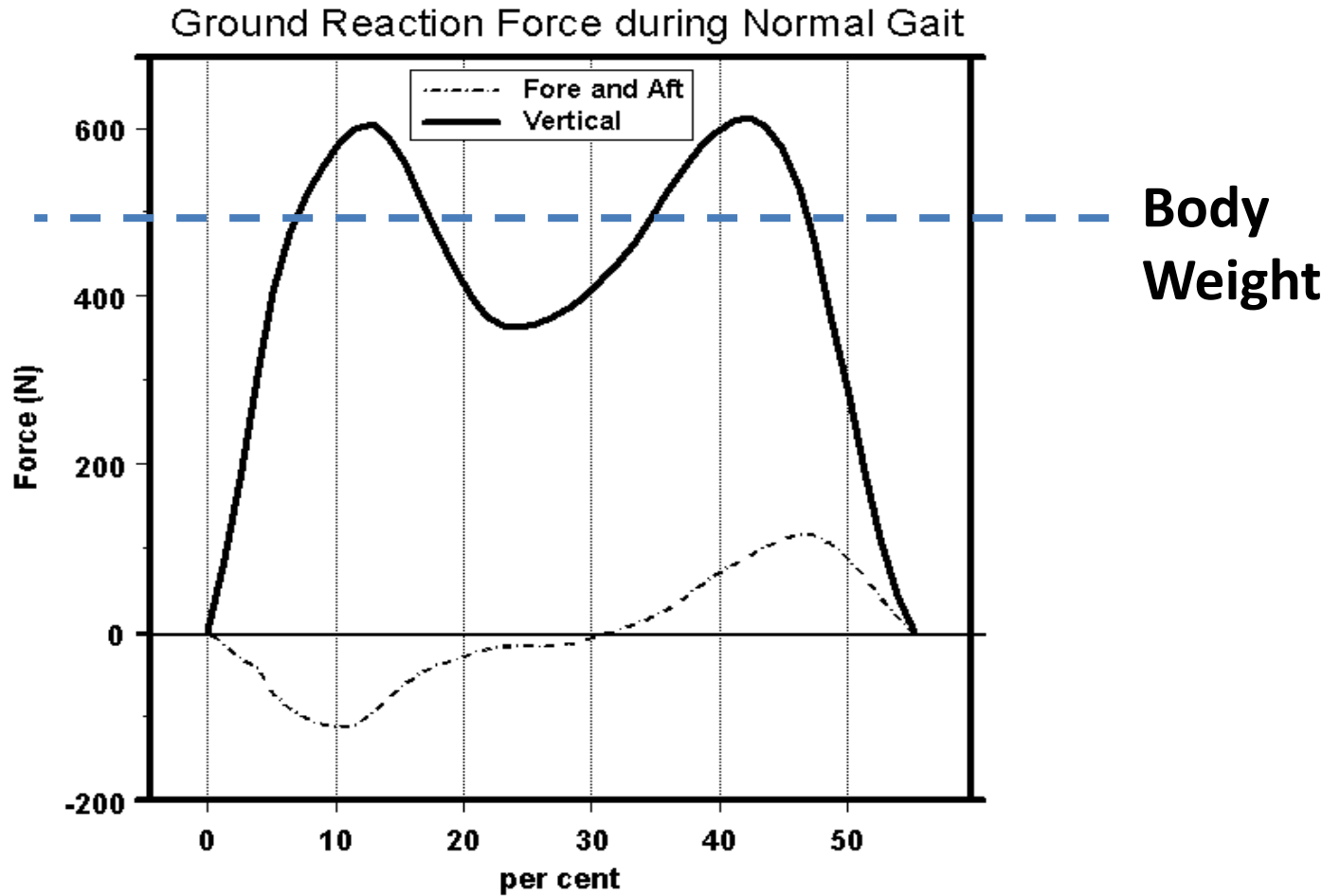
# Dynamics – consider gait



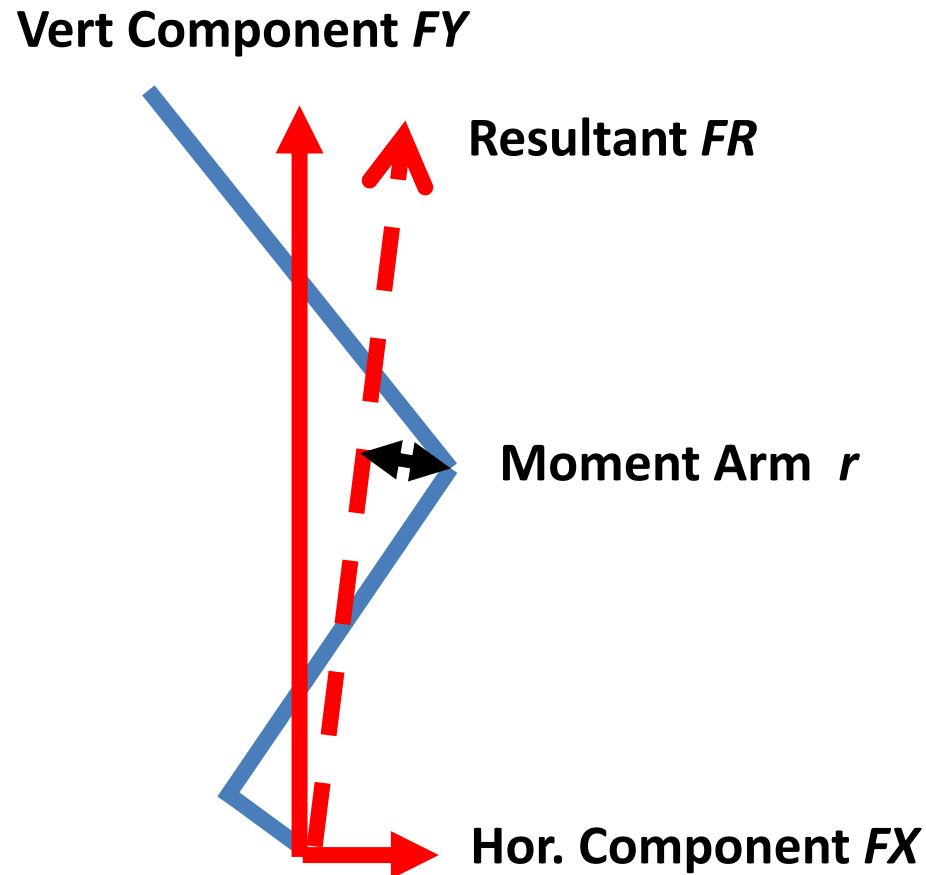
**Vertical displacement of centre of mass**

**According to Newton's Second Law of Motion:  
Vertical force = mass \* acceleration**

# Force resulting from acceleration in gait

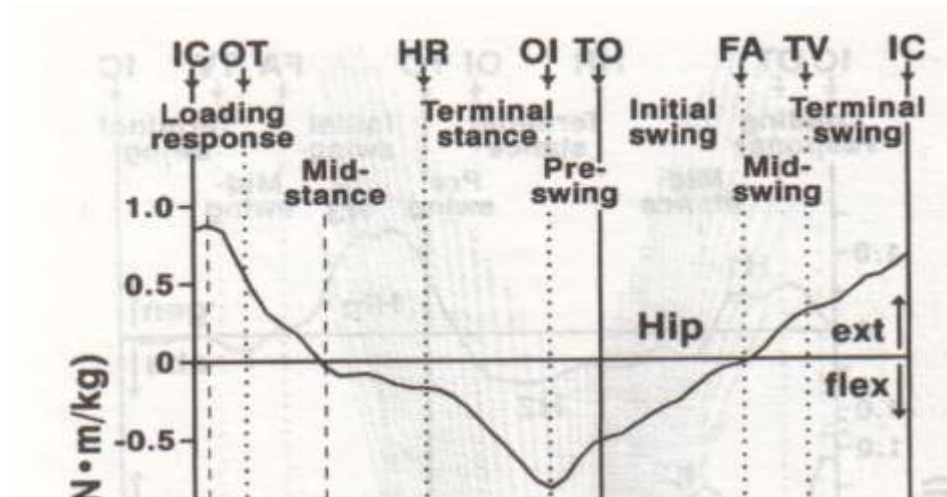


# Moments at Joints during gait

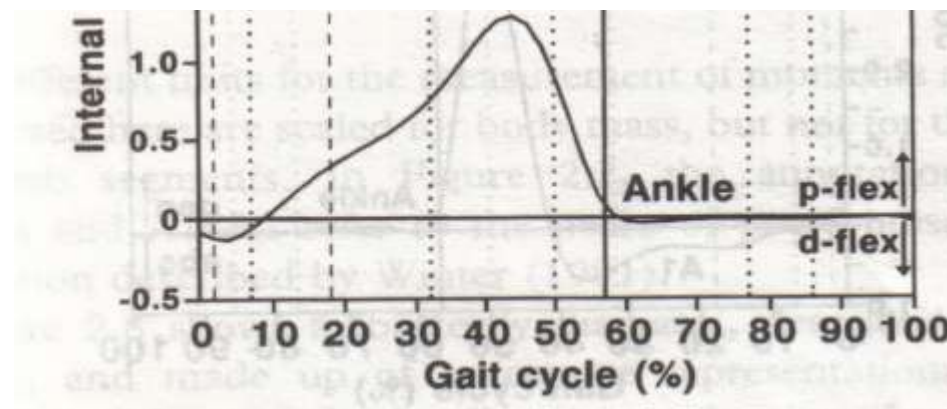


$$\text{Moment at Knee} = FR * r$$

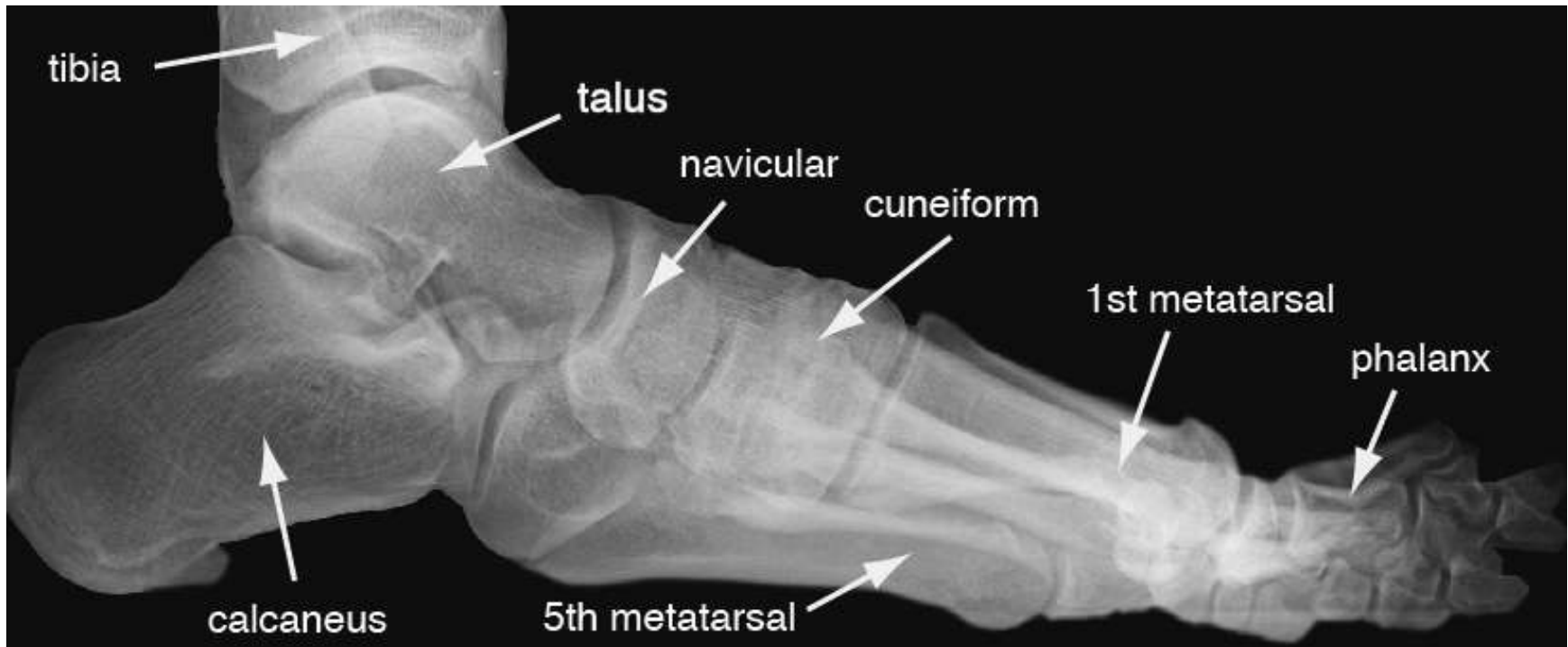
# Moments at Lower Limb during Gait



**N.B It is these moments which lead to the muscle and joint forces**



# An example of using biomechanical analysis – A Foot Model



**An on-going study with Mr Peter Briggs**

# A simple biomechanical model of the plantar aponeurosis

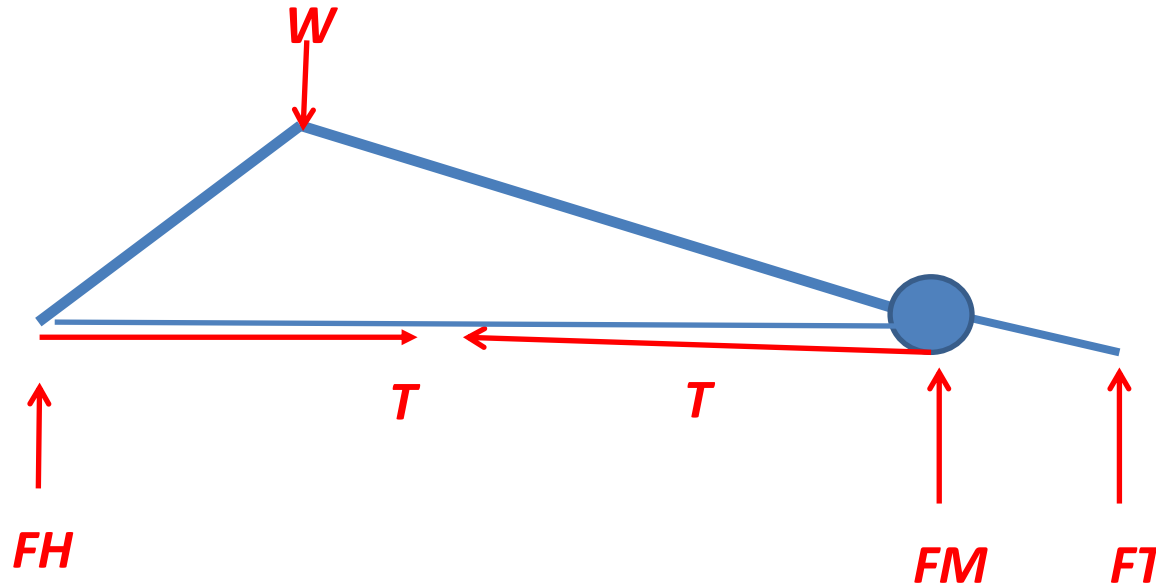
- A two dimensional model to explore basic biomechanics
  - Assumptions
    - Single ray without intermediate joint
    - Dimensions and moment arms estimated from planar radiograph
- Identify degrees of freedom and equilibrium conditions
- Look at toe off, forefoot or metatarsal head unsupported
- Identify muscle actions required

# Developing a model of the foot – how many degrees of freedom?

Simplest Model – rigid body in 2D space – 3 degrees of freedom  
Equilibrium requires 3 forces

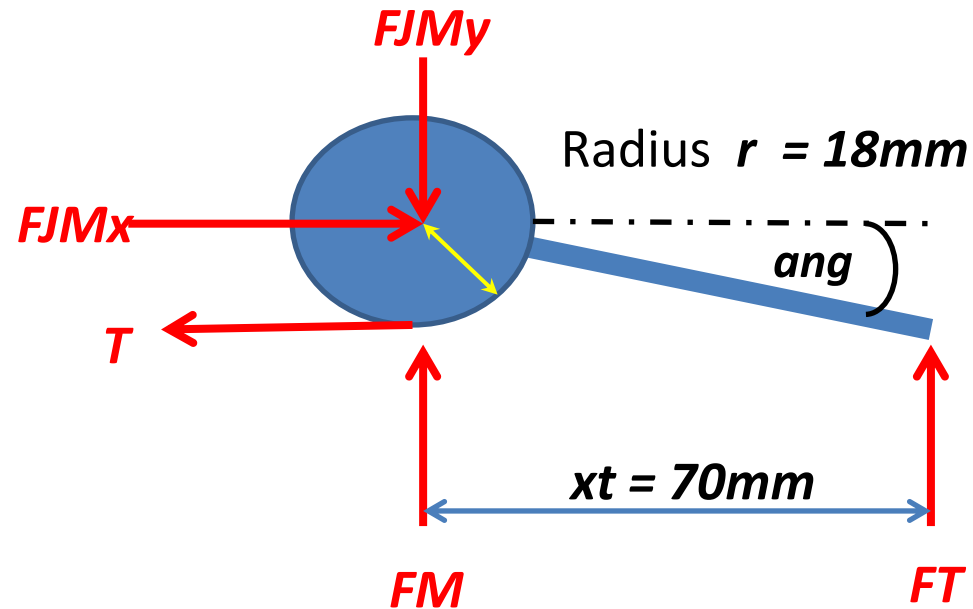
Hicks (1954) Model – Arch constrained by ligamentous structure attached to metatarsal

5 degrees of freedom + 1 constraint (aponeurosis) = 4 dof  
Equilibrium requires 4 external forces + 1 internal tension  
N.B. This is a MECHANISM



# Kinematics – How does mechanism move? – Reverse Windlass

## Mechanism and Dimensions at Metatarsal Head

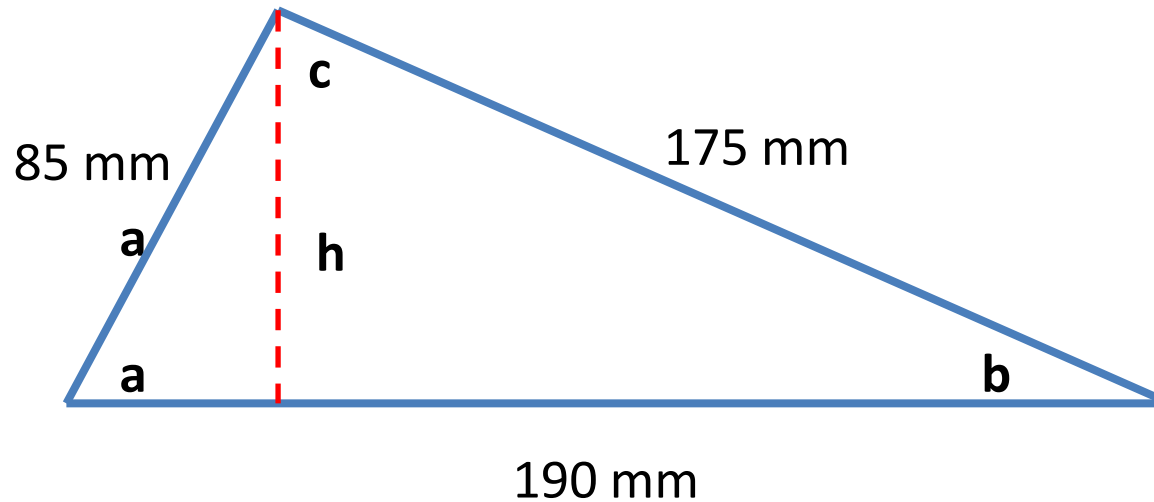


$$\text{Change in length} = r * ang$$

**N.B** Radius is small – change of length small!!

# Basic Geometry and Kinematics

## Change of arch height ( $h$ ) caused by shortening of aponeurosis



### Initial angles

$$a = 67\text{ deg}$$

$$b = 26\text{ deg}$$

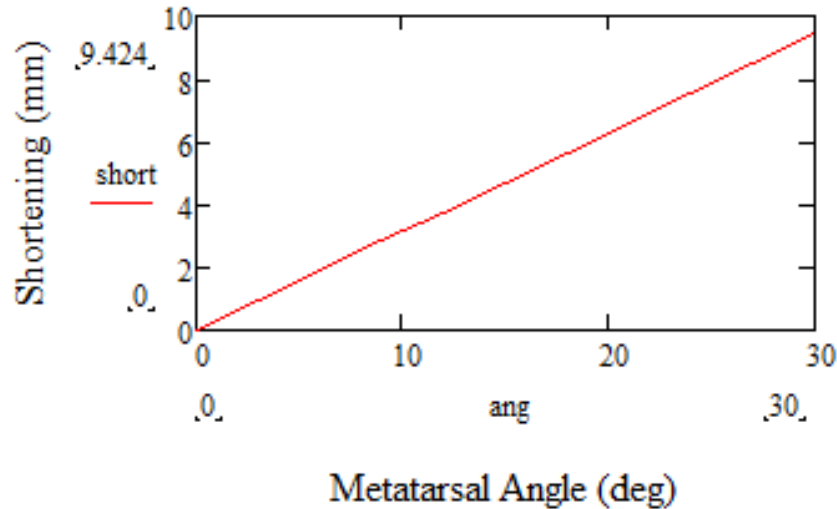
$$c = 87\text{ deg}$$

$$\text{Initial arch height} = 78\text{ mm}$$

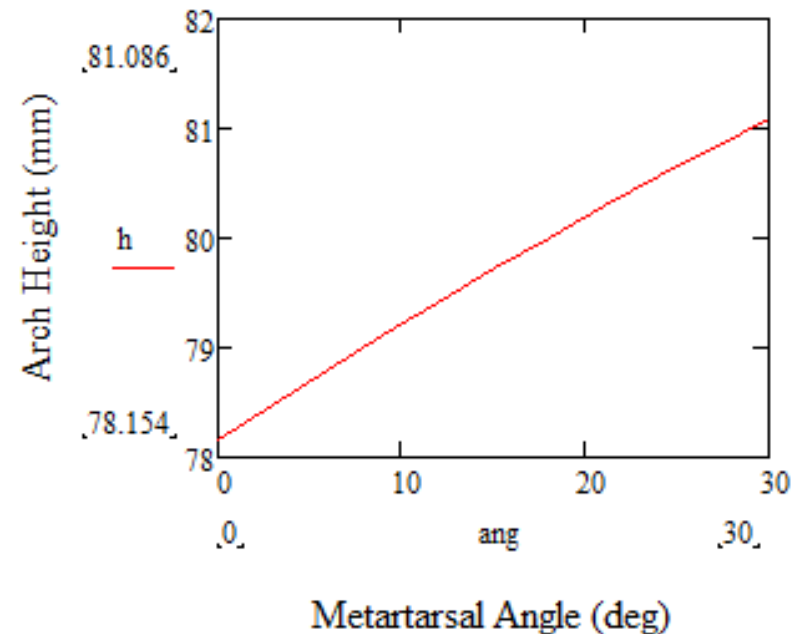
# Kinematics

## Change of arch height with shortening of aponeurosis

Change in length of aponeurosis



Change of Arch Height

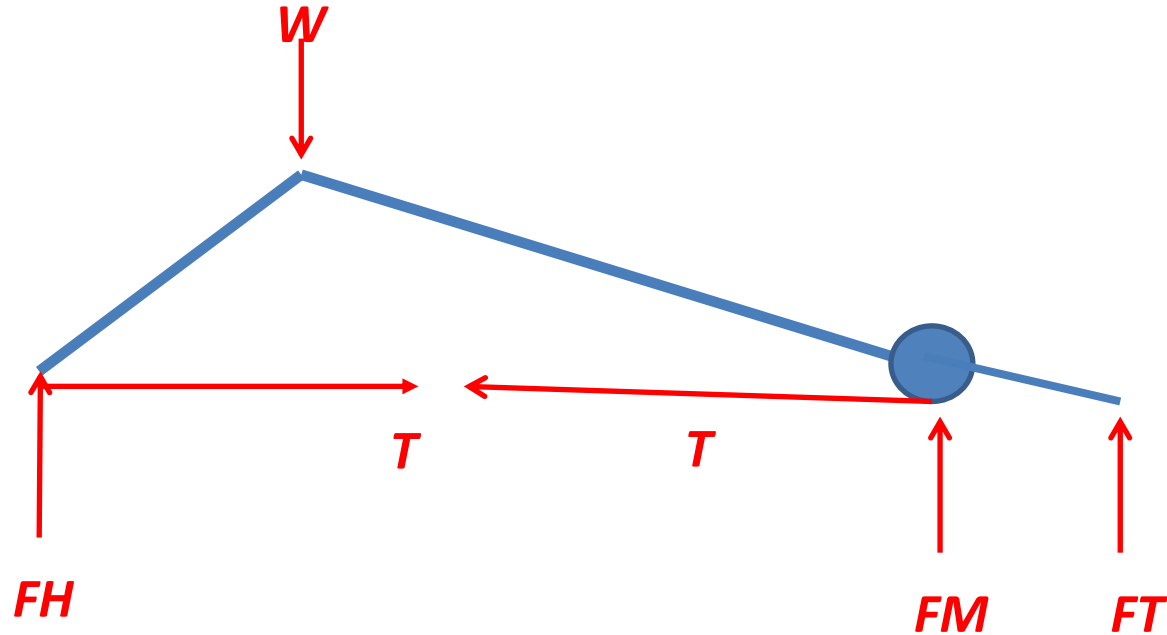


## Change in Arch Height

3 mm for 30 deg rotation!

# FORCES AND EQUILIBRIUM

## Free Body Diagram - External Equilibrium

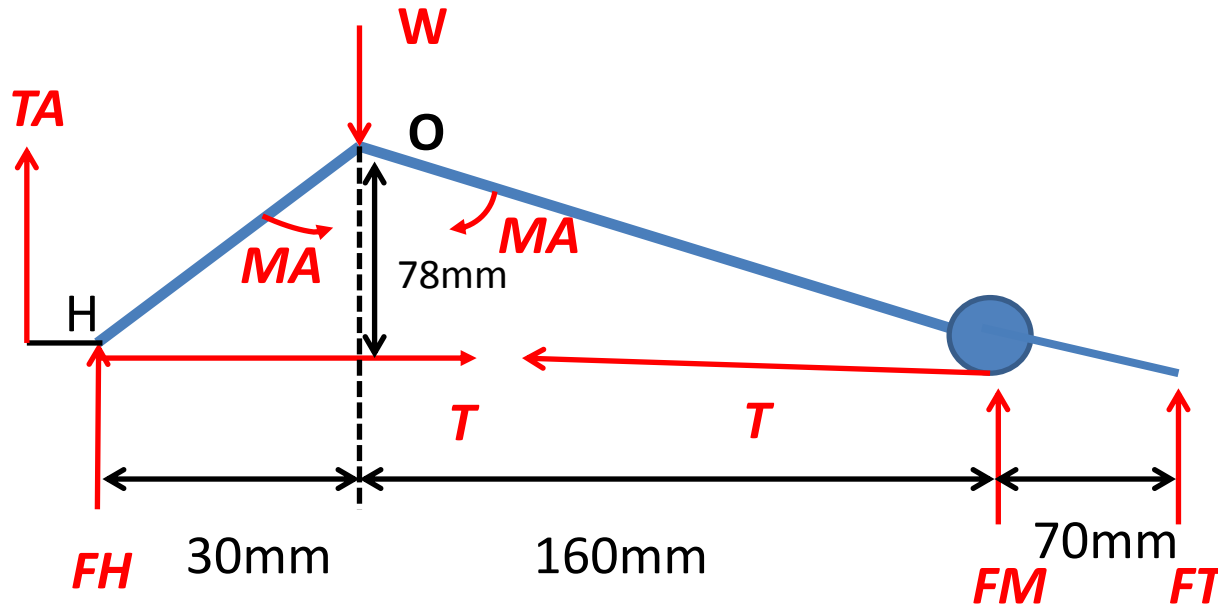


### Initial Assumptions

Quiet Standing – no muscle activity

No friction – only vertical ground reaction forces

## Equilibrium of level standing – 3 point support



### External Equilibrium

$$\text{Vertical: } FH + FM + FT - W = 0$$

$$\text{Moments about O: } 0.23*FT + 0.16*FM - 0.03*FH - .05*TA = 0$$

External equilibrium appears indeterminate! BUT consider internal mechanics

$$\text{Moments about O for OH: } .078*T - .03*FH - .05*TA + MA = 0$$

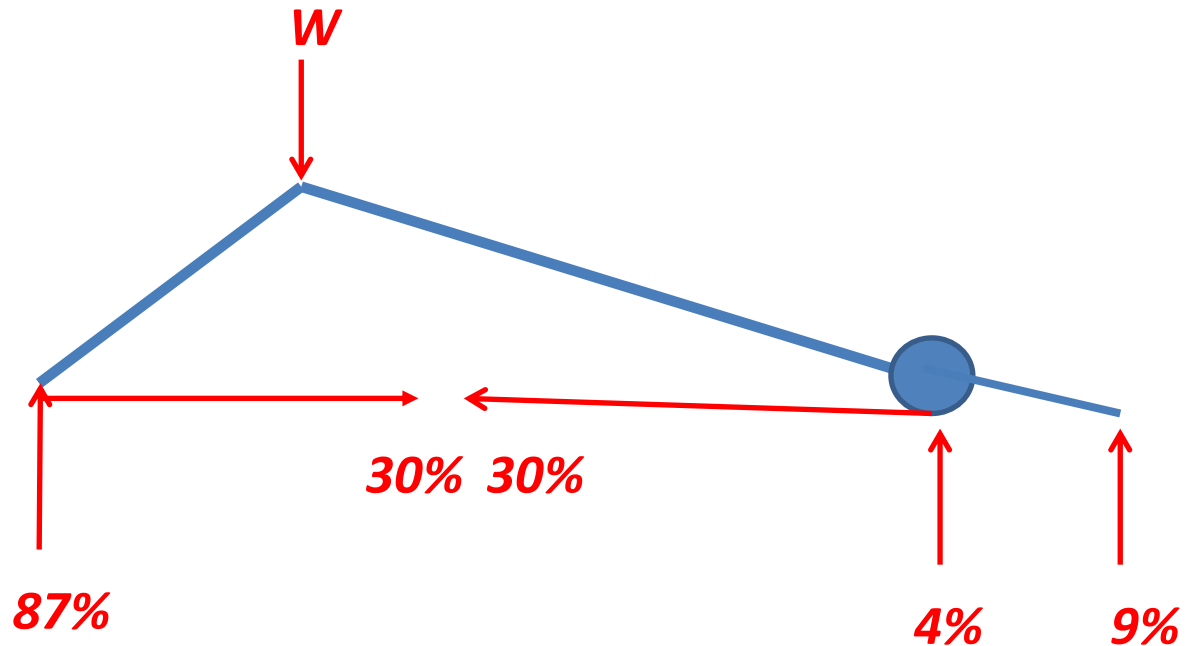
$$\text{Moments for metatarsal: } .07*FT - T*.018 = 0$$

We are dealing with a **mechanism** rather than a **structure**. Equilibrium possible with additional constraints – **heel, metatarsal heads and toe all supported**

# Equilibrium of level standing – 3 point support

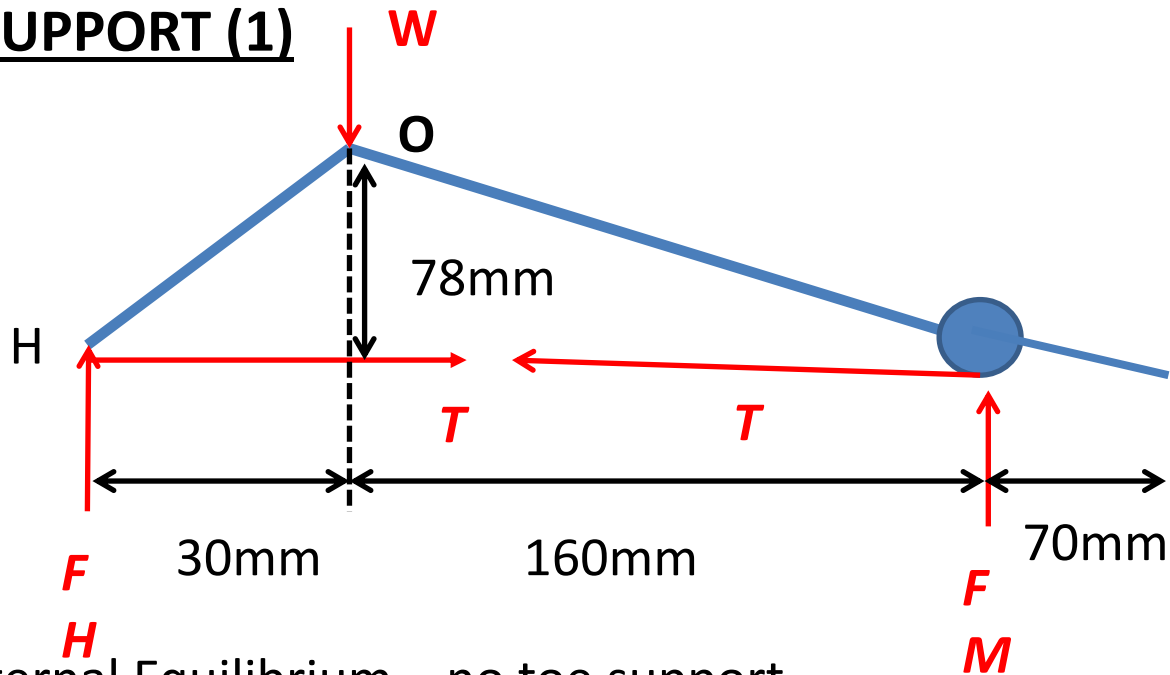
Assuming :- Tendo achilles zero tension and zero passive moment

Predicted load distribution – percentage of total supported load



**N.B.** Toe reaction force in equilibrium with aponeurosis force – moments at metatarsal head

## NO TOE SUPPORT (1)



External Equilibrium – no toe support

Vertical:  $F_H + F_M - W = 0$

Moments about O:  $0.16 * F_M - 0.03 * F_H = 0$

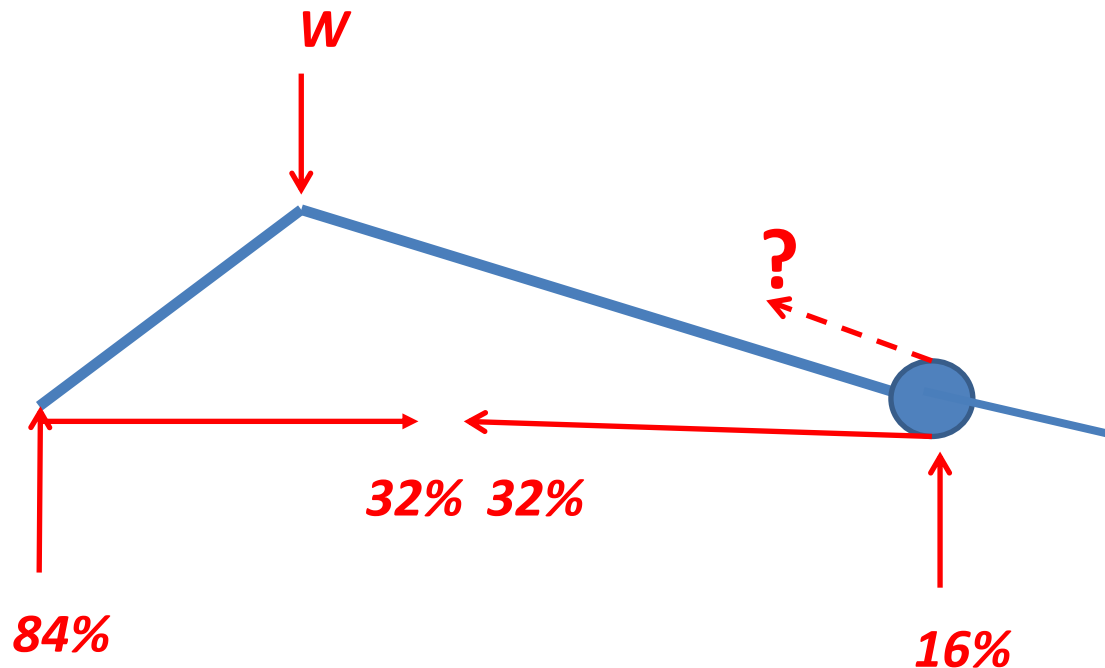
External equilibrium appears determinate! BUT consider internal mechanics

Moments about O for OH:  $.078 * T - .03 * F_H = 0$

Moments for metatarsal: **NO EQUATION**

3 UNKNOWNNS and 3 EQUATIONS - OK? NO – it's a mechanism!

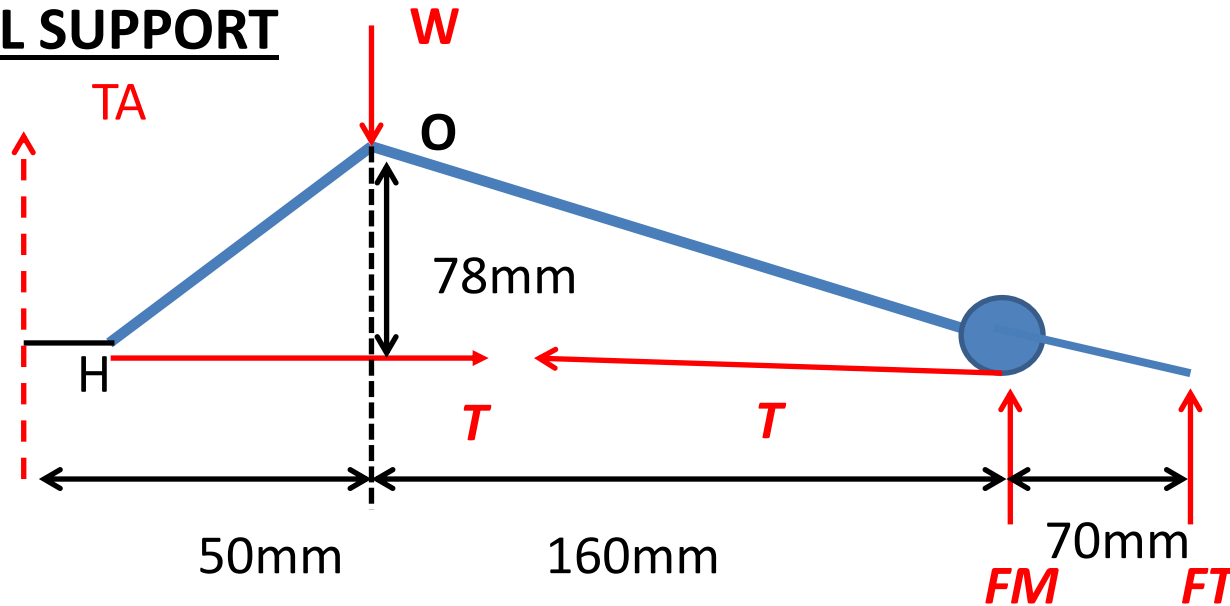
## NO TOE SUPPORT (2)



Equilibrium of arch is satisfied but metatarsal not in equilibrium!  
Need extensor muscle?? – extensor hallucis longus.

**N.B. This implies that a toe extensor is responsible for maintain arch equilibrium**

## NO HEEL SUPPORT



### External Equilibrium

Vertical equilibrium:  $FM + FT - W = 0$

Moments about O for OH:  $.078 * T = 0$  !!!

Moments for metatarsal:  $.07 * FT - 0.018 * T = 0$

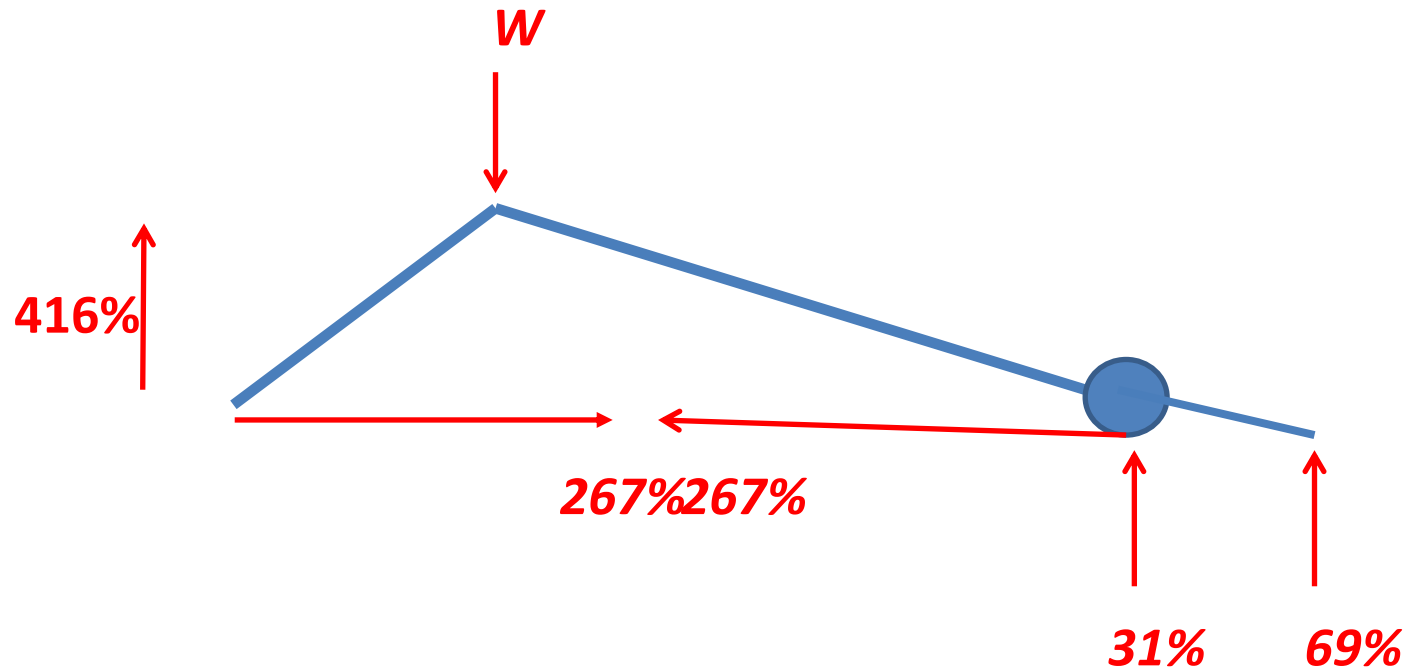
Predicting zero tension and, therefore, zero toe force, but then, from equation 2,  $FM = 0$  i.e. not in equilibrium. Need Achilles Tendon!!

N.B. TA tension does not affect vertical equilibrium (reacted by ankle contact force) but produces a couple =  $TA * 0.05$  (published moment arm).

## NO HEEL SUPPORT - continued

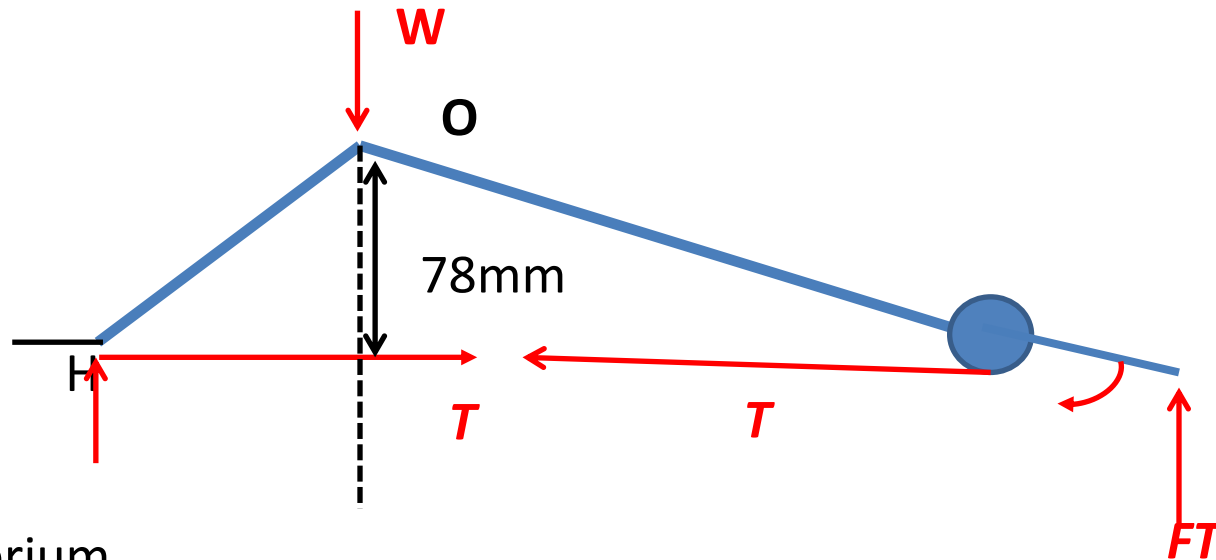
External moments:  $0.03*TA - 0.16*FM - 0.23*FT =$

New moments equation for OH:  $0.03*TA - 0.078*T = 0$



Predictions of TA and Aponeurosis forces are high – but see later!!  
Force under toe appears vital to equilibrium - **is that what happens?**  
Calcaneus acts as sesamoid between TA and aponeurosis

## NO MET HEAD SUPPORT



### Equilibrium

1. Vertical equilibrium:  $FH + FT - W = 0$
2. Moments about O for foot:  $FH \cdot .03 - FT \cdot .23 = 0$
3. Moments about O for OH:  $.078 \cdot T - FH \cdot .03 = 0$
4. Moments for metatarsal:  $.07 \cdot FT - 0.018 \cdot T = 0$

$$FH = 0.72 \cdot W \quad \text{and} \quad FT = 0.82 \cdot W$$

BUT: Equations 3 & 4 are in conflict – NO SOLUTION – this is an underconstrained mechanism!

**A short toe flexor would be required for equilibrium**

# Biomechanics of Plantar Aponeurosis - Conclusions

- The Hicks Model represents a 4 dof mechanism
- Equilibrium of the mechanism requires 3 ground reaction forces for level standing – but no muscles
- Kinematics - The predicted arch lift is small (c 3mm) – probably impossible to measure reliably
- Any stance on two contact points requires muscle activity
- The toe mechanism is vital to arch stability
- The TA load at toe off is determined by muscle or passive loads which may transfer load from the toe end to the metatarsal head.
- Predicted loads seem broadly in agreement with one relevant published study.

# Overall Comments

- This has been a small introduction to a big topic!
- Basic mechanics allows us to understand a vast amount about how joints and muscles function both in health or after injury or pathology
- In the next lecture, we'll look at a more advanced application in the design of a shoulder replacement