MINI-SYMPOSIUM: BIOMECHANICS FOR THE FRCS ORTH EXAM

(ii) Deformation of materials

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Summary
This article provides an overview of the mechanical properties of materials with particular emphasis placed upon orthopaedic biomaterials. Engineering concepts of stress, strain, shear, yielding and failure are introduced and through consideration of microstructural mechanisms, the behaviour of materials under loading is discussed. In providing a concise overview of the deformation of materials, the reader is introduced to fundamental concepts in engineering and materials science.
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Introduction

Understanding how and why materials deform is of fundamental importance in general engineering and bioengineering design. Such knowledge underpins appropriate material selection for devices and components ranging from hip prostheses to suture materials. This article outlines the underlying principles behind the material characteristics familiar to us, such as why alumina is brittle, stainless steel is tough and ductile and why polymers have low service temperatures. The starting point lies in consideration of the bonding prevalent at atomic and molecular level in the different classes of materials, from which the mechanical properties can be explained and understood. This article reviews these fundamental concepts and provides the reader with an overview of the topic that brings together elements of engineering, materials science and biomaterials.

Force and displacement

According to Newton's third law, every action has an equal and opposite reaction. Practically, when a solid is loaded by application of a force, it will respond to the applied load by displacement on the atomic scale. If this atomic displacement is significant, a macroscopic change in dimension will result. The magnitude of deformation realised is dependent both upon the applied load and the way in which the atoms that constitute the solid are held together, i.e. the atomic bonding.

The ease of atomic displacement on loading is governed by the strength of the bonds making up the material—the higher the bond strength, the greater the load that must be applied to realise a given bond separation. Figure 1 shows the typical relationship between interatomic distance, x, and potential energy for two idealised atoms joined by an interatomic bond. Such bond-potential curves are qualitatively similar for all types of bonding, both primary (ionic, covalent and metallic) and secondary (hydrogen and van der Waals). Long-range attractive interactions grow stronger as the atoms/ions approach whereas short-range strong repulsive interactions keep matter from collapsing in upon itself.

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Superposition of attraction and repulsion produces a minimum in potential energy that specifies the equilibrium separation, or the bond length, x_0. Pulling or pushing the atoms from x_0, requires energy, and thus the potential energy of the atom-bond construct rises for values of x greater or less than x_0. Mathematically, the 2nd derivative of the bond-potential curve, evaluated at the minimum, tells us how much force is necessary to stretch the bond by a small amount and this is a measure of the stiffness of the material. Therefore, the greater the curvature at the bond-potential curve minimum, the higher is the stiffness (or more correctly the elastic modulus), of the material. Beyond these small, elastic separations, the bond strength also influences the temperature at which materials melt. We find that strongly bonded solids are characterised by high melting points and high elastic moduli, whereas materials bonded via secondary bonds, for example many organic polymers, are characterised by low melting points and low elastic moduli (Table 1).

Quantification of the strength and stiffness of materials requires knowledge of the force distribution under loading. Pulling forces are classified as tensile and positive, and cause elongation of a piece of material in the direction of the force. Pushing forces are compressive and negative, and cause contraction of a material in the direction of the force.

Stress

Forces alone are by themselves not too helpful unless seen in the context of the material required to be subjected to such forces. Can a prosthetic ceramic femoral head withstand a force of 2 kN?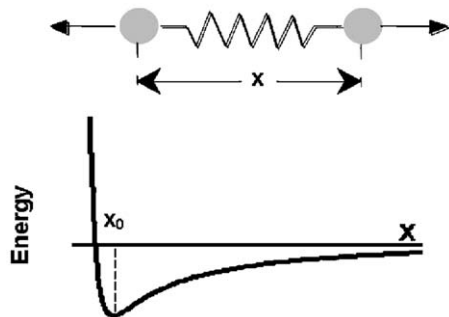 The answer may be yes if the force is spread over a large enough area, but no if the force is very concentrated. The concept of force intensity, i.e. force per unit area, is needed when determining failure risk. This leads to the simplest definition of stress, being the force applied to a body, divided by its cross-sectional area. Figure 2 illustrates a solid cylinder of material of original cross-sectional area A_o. Upon application of a tensile force F, the normal stress (σ) in the shaded plane is given by

$$\sigma = \frac{F}{A_o}.$$

Stress is usually expressed using the unit of the Pascal (Pa), which is equivalent to $1\,N/m^2$.

The use of stress instead of force is of fundamental significance in engineering, since it relates directly to the ability of a material to withstand the loads to which it is subjected. It may simplistically be stated that all materials have their own characteristics which determine the maximum stress they can safely withstand. In order to describe these characteristics in more appropriate detail, it is necessary to introduce the concept of strain.
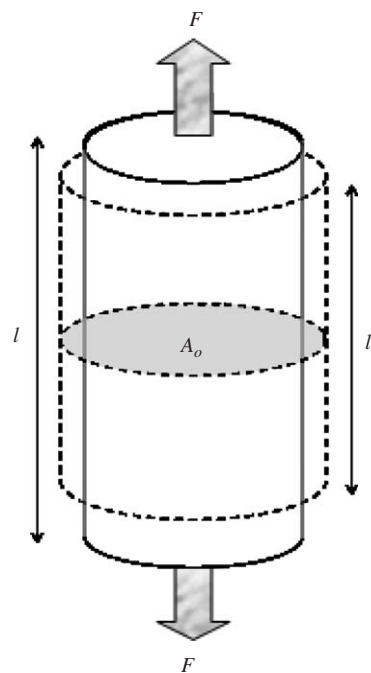

Figure 1 Interatomic bond potential energy with respect to separation distance for an idealised atom pair.

Figure 2 Axial loading of a solid cylinder of original length l_o and cross-sectional area A_o.

Table 1	Bonding-type comparisons.[1]			
Bonding type	Bond energy	Example substance	Elastic modulus/GPa	Melting temp/°C
Covalent	High	C (diamond)	1000	3500
Ionic	High	Alumina ceramic	380	2500
Metallic	Intermediate	Stainless Steel	200	1500
Van der Waals	Low	UHMWPE Polymer	1	110
Hydrogen	Low	H_2O	n/a	0

Strain

Returning to the femoral head example, knowledge of the cross-section was needed in order to determine whether or not the 2 kN force could be carried. A similar concept applies when considering the deformation undergone by materials. A 0.5 mm deformation may not be cause for concern on a tibial bone plate that was 150 mm in length, but might cause complete failure in a PMMA cement mantle that was originally only 5 mm in thickness. Normal strain, ε, is the response of the material to load, in the direction of loading, and will depend upon the applied force, cross-sectional area, length and the type of material. Strain, ε, is defined as the ratio of the change in length, $l - l_o$, of the component compared to its original length, l_o,

$$\varepsilon = \frac{l - l_o}{l_o}.$$

Since strain is defined as a length divided by a length, it has no units and is a dimensionless number. With the exception of elastomers and some polymers, for most materials and service conditions, strain will take a very small value ($\sim 10^{-4}$).

Figure 2 also illustrates the Poisson effect. Axial elongation of the cylinder is accompanied by radial contraction. Generally, any axial deformation in a material is accompanied by an associated lateral deformation of opposite sign. The Poisson's ratio of a material, v, is given by

$$v = -\frac{\varepsilon_{lateral}}{\varepsilon_{axial}}.$$

In practice, for most materials $v \approx 0.3$, with the exception of fully incompressible materials such as elastomers, for which $v = 0.5$.

Shear stress and strain

The discussion above has concentrated on forces applied perpendicularly to the section of interest, causing 'normal stress' σ and 'normal strain' ε. We also need to consider shear stresses which arise from shear forces. Figure 3 shows a cuboid 'element' of material subjected to shear forces, which cause a deformation of square faces into a parallelogram.

The shear stress, τ, is defined as the shear force, Q, divided by the shear area, A. Shear strain, γ, is defined as the angle through which the square faces deform to become parallelograms. M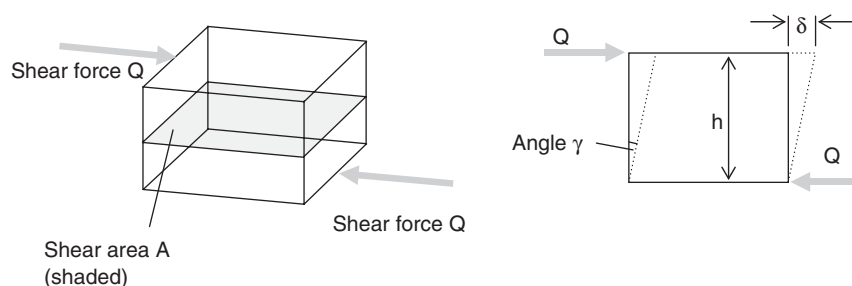ost structural components have to be designed to resist more than one type of loading as in service, axial forces, bending moments and torques may all act simultaneously. A femoral stem experiences such combined loadings during gait. Such components can be analysed by superimposing the individual, constituent stresses due to each load. The combined, resultant stresses can then be calculated and compared with the known material strength. Such analyses are the remit of stress analysis and introductory texts on this subject are given at the end of this article.[2,3]

Stress–strain behaviour of materials

Examination of the stress–strain behaviour of materials, for example via a simple uniaxial tension test, enables determination of a number of important material properties. As discussed earlier, solids will exhibit elastic deformation on loading, to a greater or lesser extent, as the macroscopic manifestation of changes in the mean interatomic separation distance, x. Plotted, there is often an initially linear relationship between stress and strain, the gradient of which is the elastic or Young's modulus. The elastic modulus, E is given by

$$E = \frac{Stress}{Strain}.$$

Unsurprisingly, the imposition of compressive, shear or torsional stresses also evokes elastic behaviour in solids. For low strains, the elastic modulus of most materials is independent of compressive or tensile loading. The shear modulus, G, is the constant of proportionality between shear stress, τ, and shear strain, γ:

$$G = \frac{\tau}{\gamma}.$$

For isotropic, linearly elastic materials of Poisson's ratio v, the shear and elastic moduli are related to each other according to the expression

$$E = 2G(1 + v).$$

Figure 4 illustrates stress–strain plots typical of many ceramic, metal and polymer materials. The differing gradients of the linear elastic regions of these curves are apparent, reflecting the differences in elastic moduli and the different types of bonding found in these materials (Table 1).
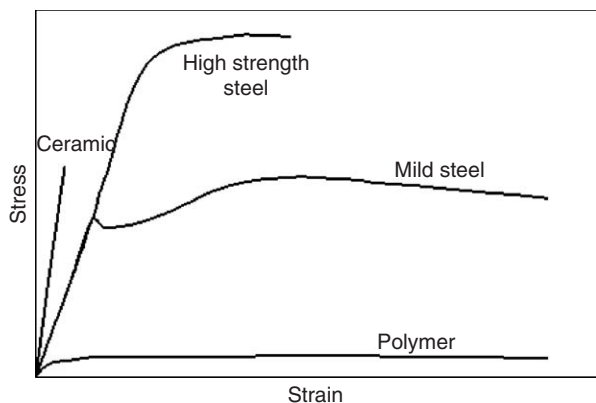
Figure 3 Shear effects.

Figure 4 Stress–strain behaviour of materials.

Yielding and failure

Beyond the linear elastic region of the stress–strain response, which typically holds only for very small strains, most materials will either fail immediately in a brittle manner, due to simple cleavage of atomic planes within the structure, or exhibit permanent, plastic deformation, which is termed ductile behaviour. Ceramics and polymers such as PMMA fail in a brittle manner and the ultimate tensile stress of such materials is the stress at fracture. In contrast, metals such as stainless steels and titanium and polymers such as UHMWPE are ductile. For these ductile materials, which are characterised by appreciable strain prior to failure, the yield point marks the transition from elastic to plastic deformation. Figure 5 illustrates the elastic to plastic transition at the yield point of a mild steel sample, the stress at which is termed the yield stress. Also shown is the ultimate tensile stress and the fracture point, which gives the fracture stress. For materials that show appreciable plastic flow prior to failure, the fracture stress calculated using the simple definition of force/original cross-sectional area produces the anomalous result of apparent falling stress with increased strain, implying a reduction in the load-carrying capacity of the material. This erroneous result arises due to the diminishing instantaneous cross-sectional area of the sample once strained beyond the ultimate tensile stress, and is a phenomenon known as necking.

Many ductile materials do not exhibit a clearly defined yield point, for example the high strength steel sample shown in Fig. 4, and for these materials the stress at a particular strain offset, for example 0.2% of strain, is used as a substitute for the yield stress in design calculations. This is also sometimes referred to as a proof stress (Fig. 6).

Work of fracture

The area under the stress strain plot is the work to fracture, and represents the total energy expended in deforming the material to failure by fracture. This is a measure of both the elastic and plastic elements of deformation and quantifies the toughness of the material. A related property is the fracture toughness of the material, which can be assessed using pendulum type impact tests (Izod or Charpy methods. See, for example, Dieter, 1998).[4]
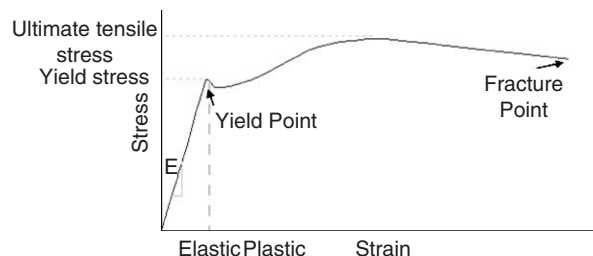

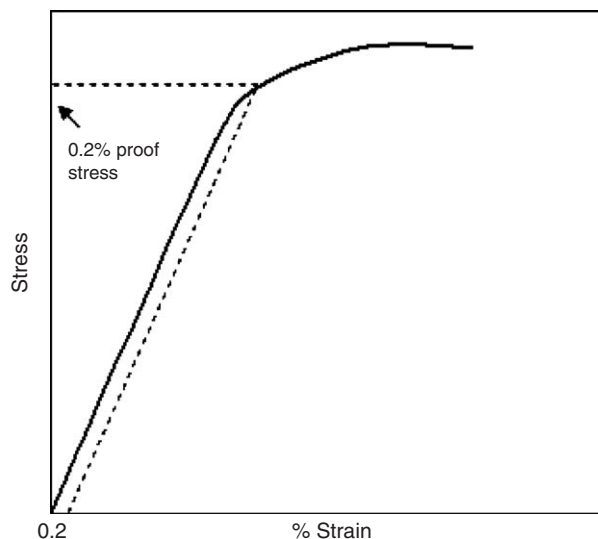Figure 5 Stress–strain behaviour of mild steel.

Figure 6 Stress strain behaviour typical of a ductile material that does not exhibit a pronounced yield point.

Unlike the elastic modulus, which for a given base alloy is insensitive to composition and micro-structural form, the work to fracture of a material is strongly influenced by material micro-structure, and can thus be controlled by appropriate thermomechanical processing. Figure 7 illustrates the typical stress strain behaviours of cast and forged specimens of a hypothetical Ti-based alloy, exhibiting similar ultimate tensile strengths and elastic moduli but quite different ductilities, indicated by the different areas under the respective stress–strain plots. The cast alloy, although of similar ultimate strength, has a much lower toughness and fails in a catastrophic manner once over-loaded. In comparison, the forged alloy has a more forgiving failure mechanism, exhibiting measurable yielding prior to fracture and in so doing providing an advanced warning of fracture failure.

Other methods of measuring ductility commonly found in material design specifications are elongation to break and the reduction in area, both of which are based upon comparisons of specimen dimensions measured before and after tensile testing. For these methods, the greater the dimensional change, the higher the ductility.

Micro-structural mechanisms of failure

The preceding discussion has highlighted generic differences between the different classes of materials based upon the bonding mechanism prevalent. Ceramics feature in

orthopaedics in a number of specific areas—Al_2O_3 and ZrO_2 are used in some joint applications as hard wearing bearing surfaces whereas hydroxyapatite is often plasma sprayed onto metallic surfaces to enhance osseointegration. Metal alloys such as 316L stainless steel, CoCrMo and Ti6Al4V are used structurally to replace bone and polymers such as PMMA and UHMWPE find application in replacement joints as bone cement and low friction bearing surfaces, respectively.

Based upon bond strengths alone (Table 1) we would perhaps expect ceramics to feature more prominently in higher strength structural in vivo applications, such as replacement joints. The reason why they do not, and metals such as stainless steels and forged Cr alloys do, is due to the differing ability of these materials to resist crack propagation. In metals, plastic deformation is facilitated by mobile lattice defects called dislocations which enable plastic flow to occur at stresses less than those needed for bulk plane cleavage.[5,6] This incremental, localised defect motion has been likened to the movement of a caterpillar (Fig. 8) and enables less energy to be expended in achieving slip (plastic deformation) than if atomic planes moved over one another en masse.

In real materials, surface and bulk defects, for example cracks and voids, are usually present. A flaw such as a crack tip acts as a stress raiser when the crack is pulled open under tension. In ductile materials, this stress raising effect is diminished by dislocation-facilitated plastic flow, which blunts the crack tip. In ceramics, glasses and other brittle materials, although dislocations exist, they are of low mobility, partly as a consequence of high interatomic bond strengths. For ceramics loaded in tension, once a crack reaches a critical length such that the stress at the crack tip exceeds the cleavage strength, the crack propagates uncontrollably, causing failure. It is for this reason that brittle materials are usually designed to be loaded in crack-closing compression.

The mobility of dislocations in metals can be controlled by thermomechanical processing to both alter the dislocation density within the metal and also to alter the micro-structure. For example, plastic deformation at temperatures of less than 1/3 of an alloy's melting temperature is termed cold work and produces work hardening via an increase in dislocation density. This increases the yield stress of the alloy as dislocation mobility falls with increasing dislocation density, due to interaction of the dislocation strain fields. Micro-structural features such as grain boundaries and second phase particles also impede the motion of dislocations and the prevalence and distribution of these features can be readily controlled during manufacture. We find that for most alloys, as the dislocation mobility falls, yield and tensile strengths rise but at a cost of reducing ductility (Fig. 9).
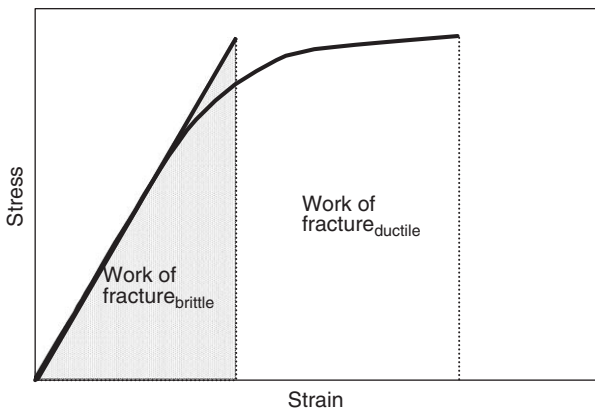

Figure 7 Stress strain plots of Ti alloys processed by casting, to produce a brittle sample and by forging, to produce a ductile sample.
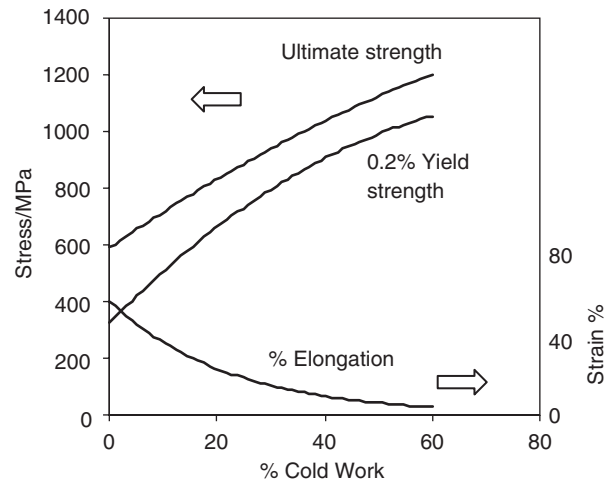

Figure 9 Effect of cold work on the yield strength, ultimate tensile strength and elongation to failure of austenitic (type 316L) stainless steel.[7]
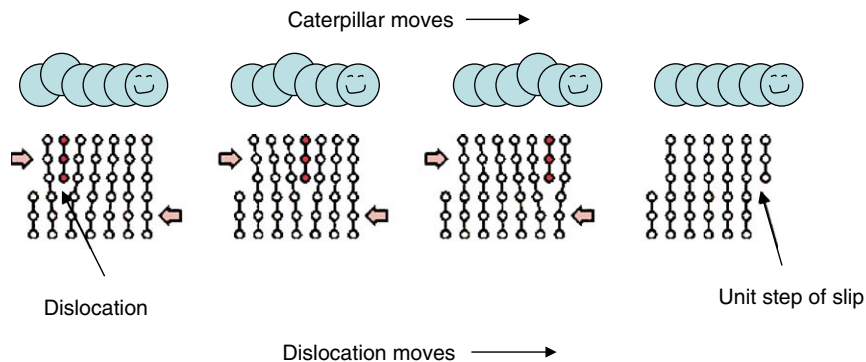

Figure 8 Representation of the analogy between a caterpillar and dislocation motion.

For ceramics, employing mechanisms that impede dislocation motion produce no useful strengthening effect, as deformation in such materials is not facilitated by plastic flow.[8] Engineered strength improvements instead rely upon reduction in flaw size by the use of manufacturing processes to minimise pre-existing defects such as cracks and pores. It is because of the high melting temperatures in these materials (Table 1) that powder processing routes, for example sintering, are often the only viable method of manufacture. Attaining a 100% dense, void free product from a sintered compact presents particular technological challenges, requiring hot isostatic pressing (HIP) during sintering and a fine grain sized starting powder. Given these conditions, high density ceramics can be produced with considerably improved toughness compared with their
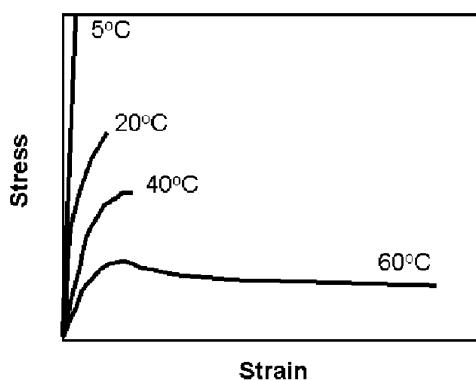
conventionally processed counterparts. Typical grain sizes of HIPed alumina used for ceramic femoral heads are $< 2\,\mu m$ which compares to the $30\,\mu m$ typical grain size for CoCrMo alloy femoral components.

For polymers, deformation generally occurs by slippage at the weaker van der Waals intermolecular bonds rather than by breakage of the stronger primary intramolecular bonds.[9] There exists a strong temperature dependence for the stability of these intermolecular bonds, with loss of strength and tendency for plastic flow at temperatures much lower than those of most metals and ceramics (Fig. 10). Thermoset polymers such as vulcanised natural rubber and silicone elastomers are characterised by some primary bond cross-links that connect the long chain molecules together. They are thus generally more stable with respect to temperature than thermoplastic polymers such as ultra high molecular weight polyethylene (UHMWPE) which have only secondary bonding between molecules. When an elastomer is loaded, the characteristic high strains are realised through molecular chain mobility, enabling uncoiling, stretching and alignment of the structure. On load removal, the cross-links provide a 'return path' for the chains to return to the pre-loaded disorganised structure. This strain-induced molecular alignment, or strain hardening, is a feature of many polymers and is a consequence of high chain mobility.

A consequence of the low strength of the intermolecular bonds in polymers is that these materials are also characterised by time dependency of the stress strain behaviour. We find that for many polymers, the measured elastic modulus depends upon rate of loading during testing, with fast loading producing the highest moduli. Additionally, appreciable plastic flow can occur when polymers are

Figure 10 Temperature dependence of the stress–strain behaviour of crystalline PMMA.

Table 2 Mechanical properties of some orthopaedic biomaterials.

Material	Elastic modulus (GPa)	Yield strength (MPa)	Ultimate tensile strength (MPa)	Strain to failure (%)
Metals				
316L stainless steel (annealed)	193	170	480	40
316L stainless steel (cold worked)	193	1200	1300	12
CoCr Mo (as cast)	210	450	655	8
CoCrMo (Hot Forged)	210	890	1400	28
Ti6Al4V	120	795	860	10
Ceramics				
Alumina >99.5%	380	n/a	350[a]	—
Hydroxyapatite[b]	50	n/a	400[a]	—
Polymers				
PMMA bone cement	2	n/a	35	7
UHMWPE	1	25	39	450
Bone				
Cortical bone	15	n/a	150	3
Cancellous bone	0.3	n/a	15	6

Data from Black (1998).[11]
[a]Compressive strength.
[b]Data from Kohn (1993).[10]

loaded at constant stress or equally, when held at constant strain, stress relaxation occurs with time. Time-dependent plastic flow is termed creep and the time dependency of the stress–strain behaviour is called viscoelasticity and is the product of both elastic and viscous (cold flow) responses of the material to loading. The viscoelastic response of a material to loading can be modelled theoretically by using combinations of elastic (spring) and viscous (dashpot) elements. The stress strain behaviours of metals and ceramics are practically independent of time and do not exhibit viscoelasticity whereas tissues such as tendons, cartilage and bone do.

Summary

Representative mechanical properties of some of the common orthopaedic biomaterials, and for comparison the mechanical properties of cortical and cancellous bone, are listed in Table 2. The derivation and quantification of the mechanical properties included in the Table have been discussed in this article. The wide range in yield, UTS and ductility of the listed alloys with respect to material condition is evident and highlights the need to be aware of the thermomechanical history of the component being used in vivo. Similar considerations can apply to ceramics and polymers. In general, material specification on the basis of material composition alone, without knowledge of the manufacturing route, is insufficient for safe design. Post-forming processes such as welding (produces locally an as-cast micro-structure), machining (can introduce stress concentrating surface defects and sub-surface stresses) and cold working (raises the dislocation density and diminishes ductility) can all materially affect the strength of a finished component and should be taken into consideration during design.

In providing a concise overview of the deformation of materials, this article has given a brief introduction to a number of engineering disciplines. Introductory texts on specific topics are referenced for those readers wishing to gain a more detailed insight than that provided by the material presented here.

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